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Statistical Modelling Of Global Solar Radiation On Horizontal Surface Using Monthly Means Daily Sunshine Hours And Some Climatic Variables For Zamfara State, Nigeria

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ABSTRACT

In this study, four empirical models on the basis of linear Angstrom- Prescotte model for predicting monthly mean daily global solar radiation on horizontal surface were developed, correlating solar radiation to climatic parameters of sunshine hours, temperature and relative humidity. The models were used to predict Global solar radiation, an average solar radiation of 16.9690 $Mjm^{-2}day^{-1}$ *and maximum and minimum of 19.2647 and 13.5222 C* ^{0} *were respectively predicted. For validation and the accuracy of the models, statistical coefficient of determination,* R^2*, mean percentage error, (MPE), mean bias error, (MBE), and root mean square error (RMSE) were used. The result showed that, model correlating solar radiation with Sunshine hours, temperature and relative humidity gives the best fit for the study location with* R^2*= 83.89%, that was;*

$$\frac{H_p}{H_o} = 0.491 + 0.188\left(\frac{n}{N}\right) + 0.02\left(\frac{T_{min}}{T_{max}}\right) - 0.341\left(\frac{RH}{100}\right)$$

Keywords: *Empirical Models, Global Solar Radiation, Climatic Parameters And Statistical Errors.*

1.0 INTRODUCTION

Solar radiation is the radiant energy emitted by the sun from continuous nuclear fusion reaction that creates energy and travels through space to the earth surface in short-wave length [1].

Solar radiation comes in many forms such as visible light, radio waves, heat (infrared), x-rays, and ultraviolet rays. Measurements of solar radiation are higher on clear, sunny day and usually low on cloudy days. When the sun is down, or there are heavy clouds blocking the sun, solar radiation is measured at zero. Solar radiation provide the earth with free, clean and inexhaustible source of energy, which if harness and utilized effectively, is of great importance to the world's high energy demand especially at this time of rising fuel costs and environmental effects such as depletion of the ozone layer and greenhouse effect [2].

This global energy crisis experienced from 1970's to date has led to intensify efforts on solar energy utilization through design and construction of solar energy systems. The comprehensive design and performance evaluation of these solar energy systems, for any particular location requires the availability of high quality global and diffuses radiation data. The best radiation data are those obtained from experimental measurements of the global and diffuse radiation components at the location under study. But due to the high cost of establishing and maintaining solar radiation measurement stations, global solar radiation in Nigeria is measured at some stations while diffuse solar radiation is not observe experimentally in any meteorological station of the country [3]. Hence several models are being developed to calculate global solar radiation using various climatic parameters. This Paper is aimed to develop equations or models for calculating Solar Radiation for Zamfara state.

Generally three classes of models for estimating radiation are in existence [4] the first class is empirical where meteorological data are used with regression techniques, the second class is based on the solar constant by allowing for depletion due to absorption and scattering by atmospheric gases, dust particles and aerosols and clouds, and the third class is based on satellites measurements of solar energy scattered and reflected into space by earth-atmosphere system.

Angstrom, [5]. developed the earliest model used for estimating global solar radiation, in which sun shine duration data and clear sky radiation (H_c) were used.

$$\frac{H_m}{H_c} = a + b * \frac{n}{N} \tag{1}$$

H_m is the monthly mean global solar radiation measured on horizontal surface ($MJm^{-2}day^{-1}$),
n is the monthly mean daily bright sun shine hours measured in hours, and
N is the maximum possible monthly mean daily sunshine in Hours, (that is, monthly average day length) while 'a' and 'b' are empirical Constants.

But this model was modified to a more convenient form by Prescott [6]. replacing clear Sky radiation H_c with extraterrestrial radiation H_o so that equation (1) becomes Angstrom-Prescott regression equation [7] ; [8] given as

$$\frac{H_m}{H_0} = a + b * \frac{n}{N} \tag{2}$$

$$K_T = \frac{H_m}{H_0} \tag{3}$$

K_T is the clearness index and H_o is the extra-terrestrial solar radiation which is theoretical.

2.0 MATERIALS AND METHOD

The input data of measured monthly mean daily sunshine hours, solar radiation and maximum and minimum temperature, relative humidity, latitude and longitude angles of the location were collected from Nigerian Meteorological Agency (NIMET) Gusau, under the Nigerian Federal Ministry of Aviation. The data obtained covered a period of 10 years (2005-2014).

Other data of extra-terrestrial radiation, declination angle, Sun set angle, maximum possible Sunshine duration (length of the day) were theoretically calculated.

2.1 Models used

The models proposed are based on the linear relationship among meteorological variables.

$$\frac{H_m}{H_o} = A_1 - B_1 \frac{n}{N} \tag{4}$$

$$\frac{H_m}{H_o} = A_2 - B_2 \frac{T_{min}}{T_{max}} \tag{5}$$

$$\frac{H_m}{H_o} = A_3 - B_3 \frac{RH}{100} \tag{6}$$

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$$\frac{H_m}{H_o} = A_4 + B_4 \frac{n}{N} + C_1 \frac{T_{min}}{T_{max} - D_1 RH} \quad (7)$$

where H_m is the measured monthly mean daily global solar radiation, H_o is the monthly mean daily extra-terrestrial radiation, n is the monthly mean daily hours of bright sunshine in hours, N is the monthly mean daily day length while *A*, *B C D* are regression constants

$$H_o = \frac{24 * 360}{\pi} I_{SC} \left[1 - 0.033\ Cos\ \frac{360\ d}{365}\right] * \left[Cos\ \theta\ Cos\ \delta\ Sin\ \sigma + \frac{\pi}{180}\ \sigma\ Sin\ \theta\ Sin\ \delta\right] \quad (8)$$

where Isc = 1367 Wm^{-2} is the solar constant , θ is the latitude angle of the location (θ = 12.170), δ is the declination angle while σis the sunset hour angle. δandσ were calculated using equation (9) and (10) respectively,

$$\delta = 23.34 \left[360 \left(\frac{284 + d}{365}\right)\right] \quad (9)$$

$$\sigma = \cos^{-1}[- \tan\theta\ \tan\delta] \quad (10)$$

Where *d* is the mean day of each month in a year, usually 15th of each month, the day of the month on which the solar declination is calculated, [7] ; [8].

2.2 Calculating regression constants.

To determine the regression constant the first order linear regression equation was employed as given below [9].

$$\mathbf{A} + \mathbf{B}x + \mathbf{Cy} + \mathbf{Dz} = K_T \quad (11)$$

Where $x = \frac{n}{N}$, $y = \frac{T_{min}}{T_{max}}$ $z = \frac{RH}{100}$ and $K_T = \frac{H_m}{H_o}$

It is an equation of least square line or first order regression. To perform the regression analysis of least square line, both sides of the Equation (11) have to be multiplied by 1, x, y and *z* successively and summing both sides to obtain equations below [9]. (Adhikari, *et al*., 2013)

$$tA + B\sum x + C\sum y + D\sum Z = \sum K_T \quad (12)$$

$$t\sum x + B\sum x^2 + C\sum xy + D\sum xZ = \sum K_T x \quad (13)$$

$$t\sum y + B\sum xy + C\sum y^2 + D\sum yZ = \sum K_T y \quad (14)$$

$$t\sum Z + B\sum xZ + C\sum yZ + D\sum Z^2 = \sum K_T Z \quad (15)$$

Where t is the number of observations i.e number of months in a year. Solving equation (12) to equation (15) yields equation (16) and (17) For one variable models, Crammers rule, for 2x2 matrixes was used to obtain the regression constants

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When $x = \frac{n}{N}$, $y = \frac{T_{min}}{T_{max}}$ $z = \frac{RH}{100}$ and $K_T = \frac{H_m}{H_o}$

$$\begin{bmatrix} t & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} A_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} \sum K_T \\ \sum x K_T \end{bmatrix} \qquad (16)$$

For the three variables model Crammers rule, for 4x4 matrixes was used to obtain the regression constants by putting the values of x, y, z and K_T in equation (17) respectively.

When $x = \frac{n}{N}$, $y = \frac{T_{min}}{T_{max}}$ $z = \frac{RH}{100}$ and $K_T = \frac{H_m}{H_o}$

$$\begin{bmatrix} t & \sum x & \sum y & \sum z \\ \sum x & \sum x^2 & \sum xy & \sum xz \\ \sum y & \sum xy & \sum y^2 & \sum yz \\ \sum z & \sum xz & \sum yz & \sum z^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} \sum K_T \\ \sum K_T x \\ \sum K_T y \\ \sum K_T z \end{bmatrix} \qquad (17)$$

2.3 Validation of the global solar radiation models (statistical analysis)

The performance or accuracy of the proposed models was tested by calculating the coefficient of determination R^2, mean bias error (*MBE*), root mean square error (*RMSE*) and mean percentage error (*MPE*). Coefficient of determination R^2 shows the linearity between the predicted and observed (measured) values. In regression, the R^2 coefficient of determination is a statistical measure of how well the regression line approximates the real data points, that is, goodness of fit of the regressor. R^2= 1 if the fit is perfect and zero when the regressor has no explanatory power whatever, and can be obtained by using the relation below:

$$R^2 = \frac{\sum\left(H_{P.i} - H_{P.a}\right)\left(H_{m,i} - H_{m,a}\right)}{\sqrt{\left(Y \sum\left(H_{P.i} - H_{P.a}\right)\right) 2YY \sum \left(H_{m,i} - H_{m,a}\right) 2Y}} \qquad (18)$$

$H_{P,i}$and $H_{m,i}$ are the ith predicted and measured values while $H_{m,a}$ and $H_{P,a}$are the average measured and predicted values of global solar radiation, and *N* is the number of observations.

MBE and *RMSE* are also worked out to check the accuracy of relationships and measure. These are defined as

$$MBE = \left[\sum \left(H_{P,i} - H_{m,i}\right) / N\right] \qquad (19)$$

$$RMSE = \left[\sum \frac{\left(H_{P.i} - H_{m,i}\right)^2}{N}\right] \qquad (20)$$

The mean percentage error is a measure of percentage deviation between the estimated and measured values or measure of accuracy of estimation computed by using the following relation

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$$MPE = \sum \left[\frac{\left(\frac{H_{m,i} - H_{P,i}}{H_{m,i}} \right)}{N} X\ 100 \right] \quad (21)$$

RSME test provides information on the short-term performance whereas *MBE* and *MPE* test provide information on the long term performance. Positive MBE indicates underestimation whereas negative one the overestimation [10]. In general, low values of root mean square error (*RMSE*), mean bias (*MBE*) and mean percentage error (*MPE*) are desirable [11].

2.4 Calculated input variables

Table 1: Calculated input variables

Month	N (Hrs) Monthly mean Day length	$\frac{n}{N}$ (Hrs) Monthly mean daily Sunshine fraction	$\frac{T_{min}}{T_{max}}$ Monthly mean daily temperature ratio	$K_T = \frac{H_m}{H_0}$ Monthly mean daily Clearness index	H_0 (MJ$m^{-2}day^{-1}$) Monthly mean daily extra-terrestrial Radiation
Jan	11.3733	0.4045	0.5139	0.5014	32.3676
Feb	11.6133	0.4339	0.5603	0.5362	34.8206
Mar	11.9333	0.4626	0.5946	0.5453	36.5458
Apr	12.0200	0.5425	0.6565	0.5038	38.5479
May	12.5600	0.5597	0.6834	0.4539	40.1180
Jun	12.6933	0.5081	0.7036	0.3820	40.7565
July	12.6235	0.5125	0.7125	0.3428	40.2862
Aug	12.3835	0.4353	0.7393	0.3503	38.8557
Sep	12.0550	0.4687	0.6949	0.4387	36.8849
Oct	11.7170	0.4890	0.6305	0.4846	34.9948
Nov	11.4371	0.4547	0.5093	0.5686	31.7646
Dec	11.3038	0.4202	0.4874	0.5023	33.2483
∑		5.6917	7.4862	5.6099	

By substituting sum of values of *x,y,z* and K_T (sum of relative sunshine hours, sum of temperature ratio, , sum of percentage relative humidity and sum of the clearness index respectively) from the table above in to equation (16) and (17), matrices were obtained. Solving the matrices by crammer's rule will give the desired regression constants.

From equation (16), three sets of regression constants were obtained as

$$\begin{bmatrix} 12 & 5.69 \\ 5.69 & 2.73 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 5.609 \\ 2.649 \end{bmatrix} A_1 = 0.672\ B_1 = 0.42$$

$$\begin{bmatrix} 12 & 74862 \\ 7.4862 & 4.7567 \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 5.6099 \\ 3.4401 \end{bmatrix} A_2 = 0.9006\ B_2 = -0.6939,$$

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$\begin{bmatrix} 12 & 4.48 \\ 4.48. & 2.1947 \end{bmatrix} \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} 5.6099 \\ 1.9301 \end{bmatrix}$ A_3 = 0.5849 B_3 = - 0.3145,

From equation (17) the regression constants obtained are:

(12 5.6917 7.4862 4.48@5.6917 2.72588 3.57957 2.174433 @7.4862 3.57957 4.75624 2.991563@4.48 2.174433 2.991563
=

$\begin{bmatrix} 5.6099 \\ 2.638356 \\ 3.426409 \\ 1.921355 \end{bmatrix}$

A_4 = 0.491 B_4 = 0.188 C_1 = 0.023 D_1 = -0.341

2.5 Proposed models

Hence the modelled equations for Gusau are:

$$\frac{H_p}{H_o} = 0.671 - 0.429\left(\frac{n}{N}\right) \quad (21)$$

$$\frac{H_p}{H_o} = 0.900 - 0.694\left(\frac{T_{min}}{T_{max}}\right) \quad (22)$$

$$\frac{H_p}{H_o} = 0.585 - 0.314\left(\frac{RH}{100}\right) \quad (23)$$

$$\frac{H_p}{H_o} = 0.491 + 0.188\left(\frac{n}{N}\right) + 0.02\left(\frac{T_{min}}{T_{max}}\right) - 0.341\left(\frac{RH}{100}\right) \quad (24)$$

3.0 RESULTS

Table 2: Measured and Predicted Monthly Mean Daily Global Solar Radiation

Month	H_m Measured GSR $Mjm^{-2}day^{-1}$	H_p Predicted global solar radiation $Mjm^{-2}day^{-1}$			
		Model 21	Model 22	Mode 23	Model 24
Jan	16.23	16.1013	17.6011	17.3026	16.9779
Feb	18.67	16.8014	17.8138	19.1613	19.0892
Mar	19.93	17.2684	17.8266	19.1914	19.2647
Apr	19.42	16.8927	17.1474	18.1824	18.7209
May	18.21	17.2848	17.0970	17.9139	18.5439
Jun	15.57	18.4623	16.7978	16.9176	17.0719
July	13.81	18.1733	16.3552	15.4557	15.5425
Aug	13.61	14.3006	15.0519	14.0517	13.5222
Sep	16.18	17.3667	15.4248	14.2483	14.0365
Oct	16.96	16.1391	16.1983	16.2864	16.3825
Nov	18.06	15.1169	17.3745	16.9803	16.9586
Dec	16.70	16.3153	18.6914	17.7734	17.5175

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Table 3: values of the statistical error of the models (2005 to 2014)

Model eqn.	R^2	MBE	MPE	RMSE
21	0.0773	-0.2606	0.06801	2.1719
22	0.6616	0.0025	-0.7064	1.5570
23	0.8251	0.0008	-0.5501	1.1098
24	0.8389	0.0019	-0.3847	1.0744

In table 3 above, the coefficients of determination (R^2) of the four proposed models varies from 0.0773 to 0.8389 (which account for 7.73 to 83.89 per cent) close to unity indicating the good fitness of the models. Model equation 24 is the best with R^2 = 83.89 per cent, followed by modelled equation (23) and equation (22) with R^2 = 82.51 and 66.16 per cent then modelled equation (21) with R^2 = 7.73, modelled (21) has the least value with R^2= 7.73 per cent. The result from equation 24, 23 and 22 implies a very good match between the measured and predicted values of global solar radiation. But only modelled equation (21) has the lowest value of R^2 and higher value of *MPE*.

*MBE*of three modelled equations are low and positive indicating slightly over estimation while only model (21) has the lowest and negative value indicating strong under estimation of the global solar radiation.

RMSE values are lowest for modelled equation (24), then equation (23), and equation (22) which indicated a very good accuracy measure of predicted global solar radiation. But for modelled equation (21), the *RMSE* is higher indicating less accuracy of the model. It was observed that the lower the *RMSE*%, the more accurate the equation used. [12]. *MPE* values for these models are lowest and negative with the exception of model equation (21) having positive value.

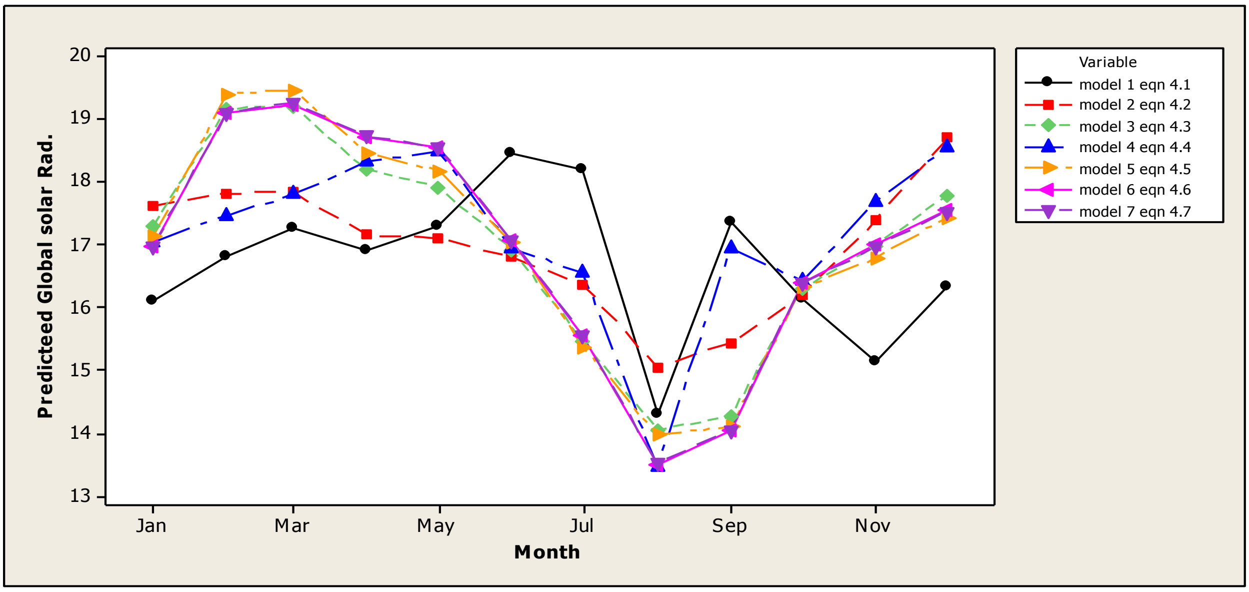

Figure 1: Monthly Variation of the Predicted Global Solar radiation

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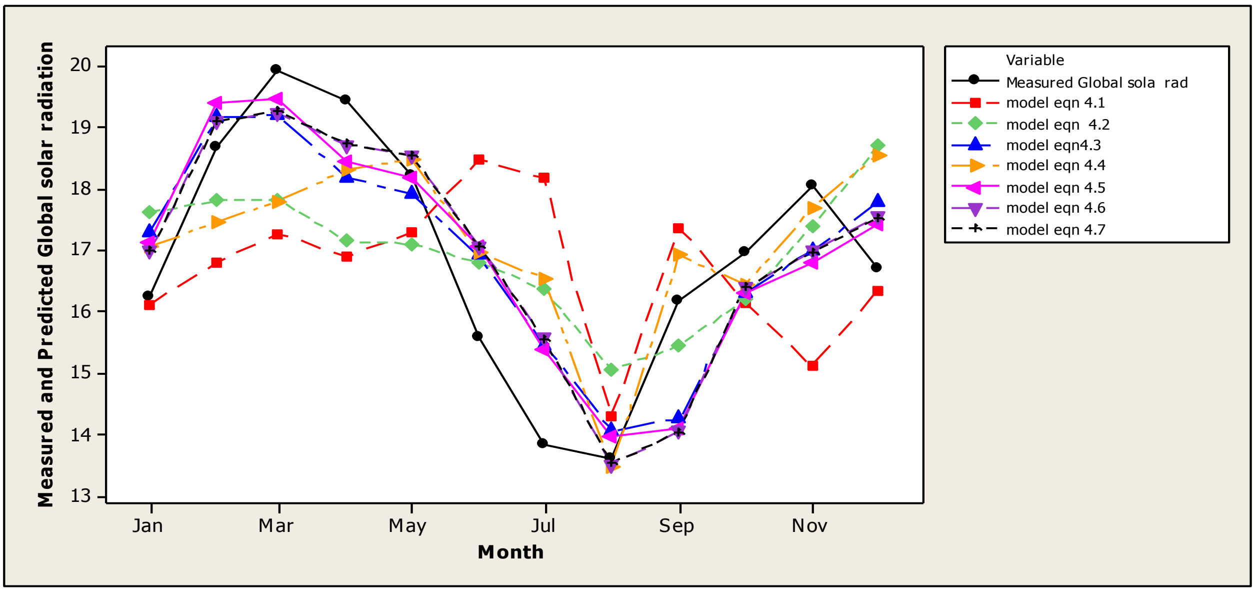

Figure 2: Comparison between Measured and predicted global solar radiation

4.0 SUMMARY

The main finding of the research may be summarized as follows**:** Four linear model equations for calculating monthly average daily global solar radiation on horizontal Surface from the sunshine ours and other meteorological variables on the basis of Angstrom- prescotte has been developed by calculating new sets of regression constants using crammer's rule. The models are expressed as a linear correlation between the global radiation and the sunshine duration, temperature ratio, and relative humidity. The performances of these models were investigated using statistical parameters, and it was found that global radiation predicted by three of these models are in good agreement with the observed or measured value, among the four models, model (24) is the best having R^2 = 83.89%. The average value of the predicted global solar radiation for Gusau city in the period of study is (from model 24) is 16.9690 $Mjm^{-2}day^{-1}$ while the maximum and minimum are 19.2647 and 13.5222 $Mjm^{-2}day^{-1}$ respectively.

5.0 CONCLUSION

It was concluded that modelled equation (24) correlating solar radiation, sunshine duration, temperature ratio, and relative humidity is the best and can be used for predicting Global solar radiation for Gusau and other areas having similar climatic conditions.

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