

# Design and analysis of Analytical Model of an Archimedes Screw Turbine

Ahmed Ibrahim Hedia<sup>1</sup>, Ehab Mina Mouris<sup>2</sup>, Omar Shehata<sup>3</sup>

Mechanical engineer<sup>1</sup>, Ain Shams University<sup>2</sup>, German University in Cairo<sup>3</sup>

Faculty of Engineering, Department of Mechanical Power

Ain Shams University

Cairo, Egypt

---

## ABSTRACT

*For centuries, Archimedean screws have been used to pump water, but mainly for that purpose. In recent decades, there has looked to be an increase in interest in using them as hydro turbines to produce electricity. While Archimedean screws are highly adapted that are used as pumps, research is currently being conducted to determine the characteristics that provide the highest performance when used as turbines. This study provides a simple but effective method for designing an Archimedean screw that may be used as a turbine. The methods for estimating the screw's outer diameter and efficiency are the most essential ones implemented in this study. The optimum of the power produced is the requirement applied for determining diameter. The diameter must be calculated by estimating the volume of the water buckets that develop between the screw blades. Although an analytical formula for this volume is nearly hard to discover, a rough approximation may be easily found. This preliminary estimate is then corrected using regression of data from turbines that are already operating at high efficiency. Finally, the rotational speed is then calculated depending on whether the rated discharge can pass through the turbine at the specified speed.*

**Key Words:** Archimedes screw, Analytical model design, Inflow head, Power and efficiency.

---

## 1. INTRODUCTION

The Archimedes screw, a device for elevating water for irrigation and drainage, is one of the oldest machineries currently in use. Archimedes (approximately 287–212 B.C.) is widely attributed with its development. Nowadays, they have found several applications in different areas like:

- Irrigation systems,
- Rain-detention dams,
- Flood-detention dams,
- Fish-conveyor systems, and
- Water sports and recreational facilities.

The usage of the screw in hydropower plants is investigated in this research. This is a new and rapidly expanding business in Europe, with more than 180 locations presently using Archimedes [1]. Water energy is critical for a long-term future since it is a clean, low-cost, and environmentally friendly source of energy [2]. Archimedean screw turbine (AST) typically has three helical blades (or flights) mounted on a long cylindrical hub (Figure 1). This AST can be covered by being installed within a permanent cylindrical tube, or it can be put in a concrete or steel trough. The AST is supported by two bearings in both circumstances. It also creates an angle with the horizontal during operation. At the AST's upper input, water rushes inside, generating buckets in the empty area between the blades. The hydrostatic forces generated by the water buckets on the blades provide a moment that rotates the blade. The water in the buckets runs down as the blades rotates, finally, leaving the screw at its bottom exit. An AST must be attached to the electrical generator using a speed-increasing gear box because of its low working speed.

Archimedean screw turbines are equivalent turbines because they run at about atmospheric pressure. Because the flow velocities inside the AST are modest, the difference in kinetic energy between the inlet and outflow may be ignored. As a result, the potential

energy of water is primarily converted into work energy by these turbines. The hydrostatic forces acting on the blades and the moments created by these forces are theoretically similar regardless of how the screw functions – as a pump or as a turbine – and, as a result, the power transmitted from or to the blades (i.e., just within the AST) is the same. As a result, the approaches for developing Archimedean screw turbines were based on screw pump design.

Nagel [3] describes such strategies in a design guidebook. Rorres [4] conducted research to determine the geometrical and operational characteristics that contribute to the Archimedean screw pump's best performance (ASP). Pumps and turbines, on the other hand, are fundamentally different in terms of the energy conversion that occurs within the AST: turbines transform hydraulic energy into work, whilst pumps do the reverse. As a result, designing screw hydro turbines only based on screw pumping methods is unlikely to result in the most efficient turbines. As a result, designing screw turbines only based on screw pumping methods is unlikely to result in the most efficient turbines. After the first screw turbine was commissioned in 1994, other design methodologies were explored and applied, resulting in a variety of designs [5].

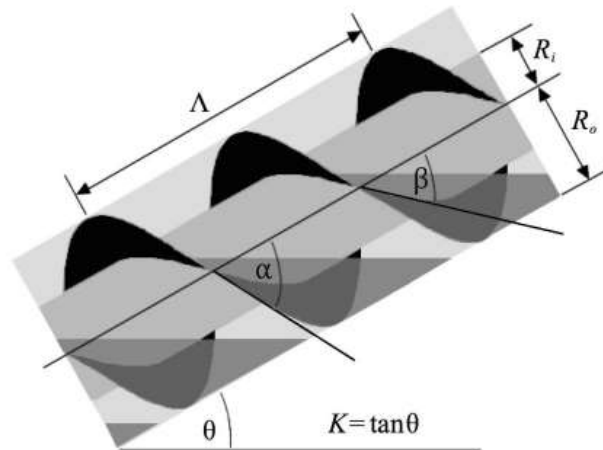


Figure 1 Side view and main parameter of AST

The aim of this paper is to obtain an analytical model for power and efficiency of AST based on certain geometric parameter when each AST bucket is optimally filled to volume ( $V_b$ ).

## 2. MAIN PARAMETERS OF AN ARCHIMEDEAN SCREW TURBINE

The geometrical parameters of an Archimedean screw turbine (AST) are directly dependent on two parameters, Internal and External parameters. Internal parameters can be selected and/or adjusted accordingly during the design process. While external parameters are dependent on the installation site, hence the designing engineer cannot adjust them. The goal of turbine design is to determine optimal internal parameter that depending on external one. (Figure 1) illustrates a schematic of an Archimedean screw that is used as a hydro turbine, as well as the key internal factors that define its shape. The external and internal parameters are shown in (Table 1).

Table 1 External and internal parameters of AST

Internal Parameters	External Parameters
$R_i$ : Inner radius (m)	$R_o$ : Outer radius (m)
$\Lambda$ : Pitch of one blade (m)	L: Screw total length (m)
$N$ : Number of blades	K: Screw slope

There are three dimensionless parameters are significant for the design of an AST:

- $\rho = \frac{R_i}{R_o}$  (Radius ratio) (1)

- $\lambda = \frac{\Lambda \tan(\beta)}{2\pi R_o}$  (Pitch ratio) (2)

- $v_U = \frac{V_U}{\pi R_o^2 \Lambda}$  (Volume ratio) (3)

There are another important parameters that shown in (Table 2).

**Table 2 Another important AST parameter**

Parameter	Description
H	AST Head (m)
Q	AST flow discharge ( $\frac{m^3}{s}$ )
n	AST speed (rpm)
$P_h$	Hydraulic power (KW)
$P_s$	Shaft power (KW)
$\eta$	AST efficiency

### 3. ANALYTICAL MODEL DESIGN

#### 3.1 Geometrical Design

The shape of inner and outer edges of AST blades are sinusoids with the same phase and period but varying amplitudes. (Figure 2) shows how certain angles are related to AST bucket's shape and calculated. Only a single bucket and the two blades of the chute containing the bucket are visible in the profiles and cross-sectional views. The following curve in the figure contains the following equations, with y denoting distance above the axis of the screw in the profile view:

- Outer edge of the lower blade:

$$y = R_o \sin(\phi) \quad (4)$$

- Inner edge of the lower blade:

$$y = R_i \sin(\phi) \quad (5)$$

- Outer edge of the upper blade:

$$y = R_o \sin(\phi - \frac{2\pi}{N}) \quad (6)$$

- Inner edge of the upper blade:

$$y = R_i \sin(\phi - \frac{2\pi}{N}) \quad (7)$$

- Water level in the bucket:

$$y = -\frac{\Lambda \tan(\beta)}{2\pi} (\phi - \phi_0) + R_i \sin(\phi_0) \quad (8)$$

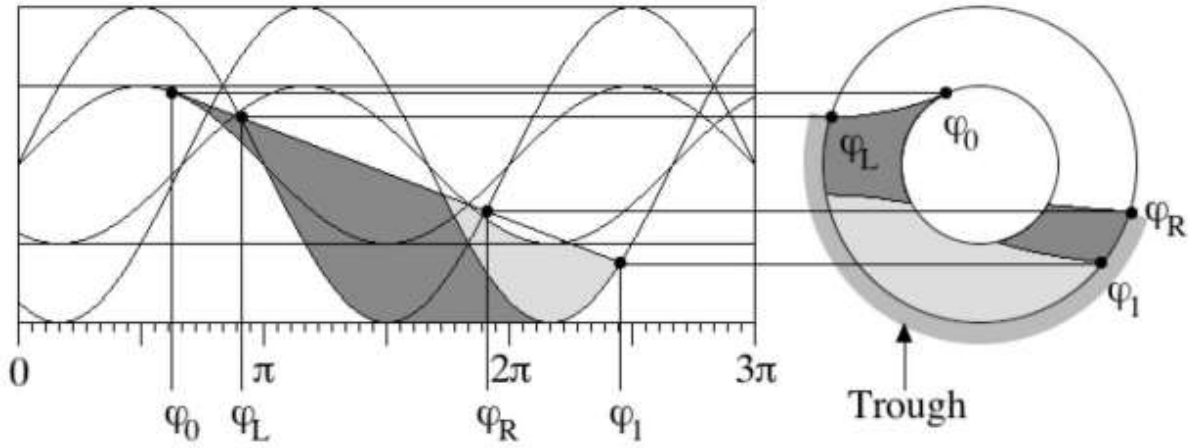


Figure 2 Coordinates determination of AST

The water level is tangent to the inner edge of the lower blade at angle ( $\phi_0$ ), where the bucket begins, and the water level meets the outside edge of the top blade for the third time at the angle  $\phi_1$  where the bucket ends. Thus,

$$\cos(\phi_0) = -\frac{\lambda}{\rho} \quad (9)$$

and,

$$R_o \sin(\phi_1 - \frac{2\pi}{N}) = -\frac{\lambda \tan(\beta)}{2\pi} (\phi_1 - \phi_0) + R_i \sin(\phi_0) \quad (10)$$

The portions of the outer cylinder in contact with the water are determined by the angles marked ( $\phi_L$ ) and ( $\phi_R$ ). They establish the minimal angular limits of the trough needed to hold the buckets of water if the outer cylinder is an open trough. Both angles are defined by the points of intersection of the water level and the lower blade's outer edge, hence they are both equation solutions

$$R_o \sin(\phi) = -\frac{\lambda \tan(\beta)}{2\pi} (\phi - \phi_0) + R_i \sin(\phi_0) \quad (11)$$

### 3.2 Inclination angle

According to the following relationship, the length of the screw,  $L$ , is determined by the turbine height and installation angle. The value of the installation angle as high as feasible is preferable from an economic standpoint, because a larger installation angle leads to a shorter screw and hence cheaper production costs. However, if the installation angle exceeds a specific amount, the screw must be put within a tube to prevent the water buckets from emptying. As a result, the turbine can get totally filled with water, and the Archimedean screw becomes an axial turbine, making it more likely that such a propeller-type turbine would be more efficient. And by the definition, the inclination slope ( $K$ ) equals to:

$$(K) = \tan(\beta) \quad (12)$$

Experimental studies conducted by Lyons [6] shown that there is no substantial improvement in peak efficiency when the installation angle goes roughly over  $25^\circ$ , while maximum power may be attained for installation angles between  $30^\circ$  and  $35^\circ$ . An installation angle of  $30^\circ$  or even  $35^\circ$  degrees can be chosen for practical reasons, such as reducing the length of the screw and ensuring installation as simple as possible while maintaining maximum efficiency and keeping the benefits of ASTs. It is worth noting that the latter figure is quite like that of Vitruvius's screws [7], which have slopes equal to  $3/4$ , equating to  $36.87^\circ$  installation angle.

### 3.3 AST length

According to the following relationship, the length of the screw ( $L$ ), is determined by the turbine height and installation angle as follow:

$$L = \frac{H}{\sin(\beta)} \quad (13)$$

### 3.4 Diameter ratio

Nagel [3] suggests diameter ratios ( $\rho$ ) between 0.45 and 0.55 based on ASP experimental findings. Rorres [5] determined during optimization research that the diameter ratio of an ASP should be about 0.54 when the number of blades ( $N$ ) ranges from 1 to 4. The increase in blade surface is followed by an increase in the hydrostatic forces acting on the blades as the diameter ratio decreases. On the one hand, when the axial component of the hydrostatic force rises, the axial thrust that forces the bearings will increase. The tangential component of the hydrostatic force, on the other hand, rises as well, but at a smaller radius ratio, thus the torque it creates is not predicted to rise considerably. The second point is corroborated by Lyons [6] experimental results for ASTs, which show that peak torque rises as diameter ratio lowers, but that the increase is only minimal for diameter ratios below 0.5. The results derived for Archimedean screws working as pumps and turbines are consistent, indicating that a diameter ratio in the range of 0.5 to 0.55 would be an acceptable situation. Rorres [5] ideal ratio ( $\rho = 0.54$ ) for ASPs appears to be appropriate for ASTs as well.

### 3.5 Bucket volume

The computation of the volume ( $V_b$ ) is a critical issue that occurs while developing an AST. This volume is normally highly dependent on the turbine discharge as well as the blade pitch and turbine speed selected. When the blade pitch and turbine speed are set, the bucket volume is influenced by five surfaces (Figure 2):

- The free surface of the water in the bucket, which creates the installation angle with the turbine axis,
- The wetted surface of the downstream blade,
- The wetted surface of the upstream blade, which is small,
- The wetted inner surface of the trough, and
- The wetted outer surface of the hub.

Because the blades are characterized by sinusoids, it is impossible to derive an analytical expression for

the true volume of the water bucket, not only because it has a convoluted form, but also because the equations to be solved are transcendental. As a result, Dragomirescu [8] advise using an approximate but straightforward approximation of this volume. After adjusting the estimate with a correction factor, a feasible formula for the outside diameter may be found. The correction factor can be determined using regression analysis of experimental data or data from high-efficiency turbines. Now, the AST bucket volume ( $V_b$ ) can be written as:

$$V_b = \frac{2k_v R o^3 - R i^3}{3 K} = \frac{k_v D_o^3}{12 K} (1 - \rho^3) \quad (14)$$

where ( $k_v$ ) is a correction factor for the estimation that was made. It is also important to notice that ( $k_v$ ) must be less than 1. The dimensionless parameter that related to bucket volume is called volume ratio  $v_U$ . This parameter describes the ratio between the water volume of  $N$  buckets ( $v_U$ ) to the total volume of AST and described in equation (14).

### 3.6 Outer diameter

The turbine speed is significantly connected to the selection of an Archimedean screw's outer diameter ( $D_o$ ). Nagel [3] suggests the value of the speed ( $n$ ) in rpm should not exceed the upper limit, according to this equation:

$$n = \frac{50}{D_o^{\frac{2}{3}}} \quad (15)$$

The speed restriction comes from the need to prevent water spilling outside the screw turbine during normal operation, which might result in an unwanted and uncontrolled emptying of the water buckets. The turbine speed, on the other hand, should be selected such that the flow rate absorbed by the screw turbine during a full revolution is equal to the discharge. The following relationship equation may be calculated assuming that number of buckets ( $N$ ) are filled with water throughout a full rotation:

$$n = \frac{60Q}{NV_b} \quad (16)$$

Where ( $Q$ ) is water flow. Equation (17) is obtained by substituting ( $n$ ) from equation (15) and ( $V_b$ ) from Equation (14) and get:

$$n = \frac{50}{D_o^{\frac{2}{3}}} = \frac{720KQ}{k_v N D_o^3 (1 - \rho^3)} \quad (17)$$

So, the exertion of the outer diameter ( $D_o$ ) results to:

$$D_o = \left[ \frac{14.4KQ}{k_v N(1-\rho^3)} \right]^{\frac{3}{7}} \quad (18)$$

### 3.7 Blade pitch

ASPs and ASTs are fundamentally different, as stated in the introduction. In the case of ASPs, optimal design refers to the shape that optimizes the volume of water supplied in a single screw Rorres [5]. The ideal design in the case of ASTs refers to the shape that optimizes the power extracted. Because the turbine speed is restricted, maximizing the power necessitates maximizing the torque and, as a result, the tangential component of the hydrostatic force. At the same time, an AST's pitch ( $\Lambda$ ) should be kept within specific parameters. Dragomirescu [8] suggests that the pitch ( $\Lambda$ ) of an AST be chosen so that the free surface in a bucket meets the downstream blade on the coordinate plane, which contains the turbine axis, while the upstream blade is only lightly wetted (Figure 3). The negative moment produced by the hydrostatic forces on the blades (i.e., the moment opposing the screw rotation) is practically negligible.

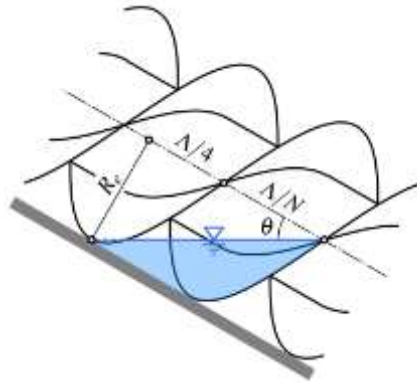


Figure 3 Side projection of the resulting free surface of the water in a bucket.

It is worth noting that the moment created on the lower half of the upstream blade (below the turbine axis) is fully negative, as is the section of the downstream blade's top half (i.e., above the turbine axis). The following connection holds true for the suggested solution:

$$\Lambda = \frac{4NR_o}{K(4+N)} = \frac{2ND_o}{K(4+N)} \quad (19)$$

The equivalent dimensionless pitch ratio ( $\lambda$ ) becomes:

$$\lambda = \frac{4N}{2\pi(4+N)} \quad (20)$$

It can be shown that, in the suggested method, the dimensionless pitch ( $\lambda$ ) is determined by the number of blades and is unaffected by screw geometry or installation angle. (Table 3) shows the results of Rorres [4] and paper equation in terms of the dimensionless pitch ( $\lambda$ ).

Table 3 Optimal dimensionless pitches recommended for Rorres's ASP and comparison with dimensionless pitches proposed in the present paper for ASTs.

N	Dimensionless pitch, $\lambda$		
	ASP, Rorres	AST, Equation	Regression %
2	0.1863	0.2122	12.21
3	0.2217	0.2122	18.74
4	0.2456	0.3183	22.84

### 3.7 Inflow head

The profile of the fluid level when water enters from a channel with a flow  $Q$  and a head  $h_1$ , which is the sum of the inflow head ( $h_{in}$ ) and the sill height ( $w$ ). Water enters the screw through Plane 2, which is perpendicular to the water flow and has a slightly different head ( $h_2$ ) than the

inflow head ( $h_{in}$ ). To calculate the value of ( $h_{in}$ ) for a given flow, an analytical model for the water depth ( $h_2$ ) must first be developed. Because the speed of the flow rises in this plane due to the downward motion of the buckets of water, the water level creates a drop-down profile after traversing Plane 2.

The dependency of volume ratio  $v_U$  on average height ( $h_3$ ), or equivalently on dimensionless height  $\kappa = \frac{h_3}{R_e}$ , is then calculated. The water level in Fig. (4, b) is cuts the central tube and derived as:

$$v_U = \frac{\alpha_8 - \alpha_9 \rho^2}{2\pi} - \frac{(1-\kappa)}{\pi} [\sqrt{1 - (1-\kappa)^2} - \sqrt{\rho^2 - (1-\kappa)^2}] \quad (21)$$

The angles in equation (21) are given by:

$$\alpha_8 = 2 \arccos(1 - \kappa) \quad (22)$$

$$\alpha_9 = 2 \arccos\left(\frac{1-\kappa}{\rho}\right) \quad (23)$$

For a fixed value of ( $v_U$ ), numerical techniques may be utilized to determine ( $\kappa$ ) from equation (21). It's also worth noting that equation (21) is affected only by the screw's radius ratio ( $\rho$ ), not its pitch ratio ( $\lambda$ ).

Next, it is assumed that the head ( $h_2$ ) is the vertical projection of the height ( $h_3$ ), i.e.,  $h_2 = h_3 \cos(\beta)$ . The water depth ( $h_{in}$ ) may then be calculated using Bernoulli's equation in Planes 1 and 2:

$$h_1 + \frac{c_1^2}{2g} = w + h_2 + \frac{c_2^2}{2g} (1 + \zeta) \quad (24)$$

where  $\zeta$  is hydraulic loss factor that water comes from the rectangular channel Plane 2 to the inclined circular entrance of the screw, it may be approximated using Borda [9] head loss and it can be written as:

$$\zeta = \left( \frac{v_U \pi R_e}{\kappa \cos(\beta) b_2} - 1 \right)^2 \quad (25)$$

Also,  $c_1$  and  $c_2$  are velocity flow at plane 1 and 2 respectively. Water flow through plane 1 is given by  $Q_1 = c_1 b_1 h_1$  and water flow through plane 2 is given by  $Q_2 = c_2 b_2 h_2$ . As indicated in (Figure. 4)  $b_1$  and  $b_2$  are the channel widths. The inflow height of a flow  $Q$  at  $b_1 = b_2$  becomes:

$$h_{in} = h_2 + \frac{1}{2g} \left( \frac{Q}{h_2 b_2} \right)^2 \left[ 1 + \zeta - \left( \frac{h_2}{h_1} \right)^2 \right] \quad (26)$$

### 3.8 Power and efficiency calculations

The difference in water levels (Figure. 5) on each blade's upstream and downstream sides generates a hydraulic force  $F_{hyd}$  that moves with the screw speed  $c_{ax}$ , Brada [10], resulting in a power:

$$P_s = F_{hyd} c_{ax} \quad (27)$$

The trough inclination angle ( $\beta$ ) relative to the horizontal and an AST pitch ( $A$ ) distance between two individual blades. The water depth increases by:

$$\Delta h = \frac{H}{N} \quad (28)$$

The hydrostatic force  $F_{hyd}$  is then calculated for a rectangular portion of unit width and determined to:

$$F_{hyd} = \frac{(h_{in} + \Delta h)^2 - h_{in}^2}{2} \rho g \quad (27)$$

Where ( $\rho$ ) is water density and ( $g$ ) is gravitational acceleration. The velocity flow ( $c_{ax}$ ) to the AST is formulated as:



$$c_{ax} = \frac{h_{in}}{h_{in} + \Delta h} c_2 \quad (28)$$

The produced power  $P_{blade} = F_{hyd} c_{ax}$  at number of blades ( $N$ ) and thus, the total power produced become  $P = P_{blade} N$  and therefor the hydrostatic power is:

$$P_{hyd} = \rho g Q H = \rho g N c_2 \Delta h h_{in} \quad (29)$$

At  $n = \frac{h_{in}}{\Delta h}$  the theoretical efficiency will be:

$$\eta_{th} = \frac{P}{P_{hyd}} = \frac{2n+1}{2n+2} \quad (30)$$

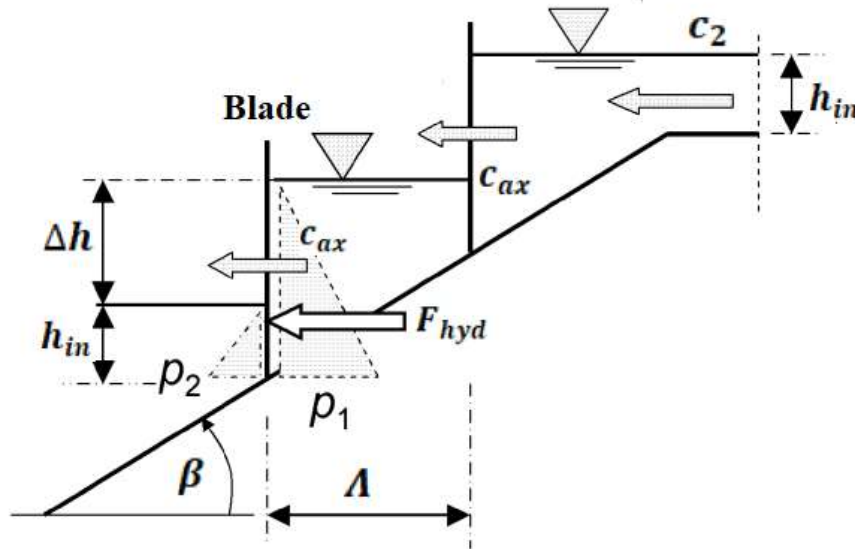


Figure 4 Forces acting on individual screw blade.

#### 4. DESIGN PROCEDURE

The previous section considerations allow for the construction of a simple AST design technique. (Figure 5) depicts this approach in the form of a flowchart. Once the turbine head and discharge are determined, the design technique is simple and allows all the parameters listed to be calculated step by step.

#### 5. CONCLUSION

The paper presents useful considerations for establishing the parameters of an Archimedes screw turbine during its design process. The internal and external parameters for AST are defined as well as the dimensionless parameters are specified. An analytical algorithm is proposed in this paper by identifying the coordinate of AST at maximum filling point. Simplified a formula to calculate the bucket volume and a correction factor that used for this approximation. However, based on bucket volume, the volume ratio is derived. Moreover, the blade pitch is derived and thus the pitch ratio is considered. A formula is used to calculate the outer diameter of AST is derived based on bucket volume calculation. Furthermore, the expression of the outer diameter is consequently utilized to come up with a formula to calculate turbine speed. The inflow head which is the effective distance of water that inters the AST is proposed. A simple way for determining the axial and tangential components of the hydrostatic forces acting on the blades of the AST would be useful of this work.



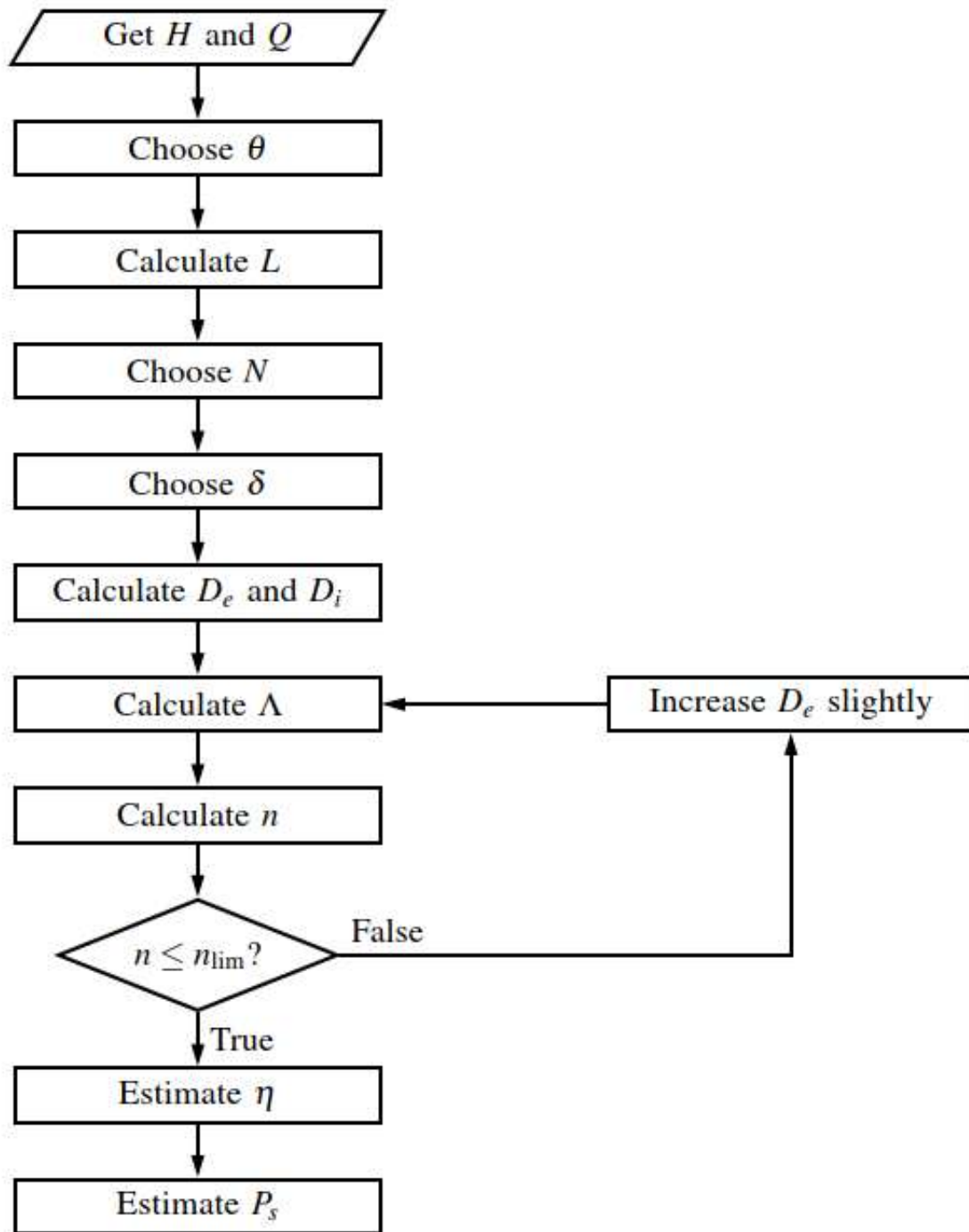


Figure 5 Design procedure for Archimedean screw turbines.

## REFERENCES

- 1- Lashofer, A., Kaltenberger, F., and Pelikan, F. (2011). “Wie gut bewährt sich die Wasserkraftschnecke in der Praxis.” *Wasserwirtschaft*, 7–8, 76–82.
- 2- Abhijit Date, Aliakbar Akbarzadeh. *Design and cost analysis of low head simple reaction hydro turbine for remote area power supply*. J Renew Energy 2009; 34:409-15.
- 3- Nagel G 1968 *Archimedean Screw Pump Handbook* (Schwäbisch Gmünd: RITZ-Pumpenfabrik OHG).
- 4- Rorres C 2000 *Journal of Hydraulic Engineering* 126 72–80.
- 5- Waters S R 2015 *Analyzing the Performance of the Archimedes Screw Turbine within Tidal Range Technologies* Master’s thesis Lancaster University Lancaster, UK.

- 6- Lyons M W K 2014 *Lab Testing and Modeling of Archimedes Screw Turbines* Master's thesis University of Guelph Guelph, Canada.
- 7- Vitruvius 1999 *Ten books on architecture* (Cambridge: Cambridge Univ. Press).
- 8- Andrei Dragomirescu 2021 *Design Considerations for an Archimedean Screw Hydro Turbine* IOP Conference Series: Earth and Environmental Science 664 (2021) 012034.
- 9- Brada, K. (1996b). "Wasserkraftschnecke—Eigenschaften und Verwendung." Proc., Sixth Int. Symp. on Heat Exchange and Renewable Energy, Szczecin, 43–52. Horch, J. C. (1916). "Proefnemingen met een watervijzel." De Ingenieur, 49, 945–954.
- 10- Brada, K. (1999). Wasserkraftschnecke ermöglicht Stromerzeugung über Kleinkraftwerke [Hydraulic screw generates electricity from micro hydropower stations]. Maschinenmarkt Würzburg, Mitteilung 14, 52 56.<http://www.maschinenmarkt.vogel.de/index.cfm?pid=5156&pk=303> [in German].