

# Comparative Study of the Performance of Different Digital Modulation Schemes

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## ABSTRACT

*The choice of a digital modulation scheme to be used in a given application depends on the most desired performance parameters in that application. Different communication set up requires different performance parameters to be set to priority. Wireless communication requires spectrum efficient modulation and demodulation schemes under limited bandwidth. Modulation scheme that requires the least power to transfer information are also desirable as well as modulation schemes that offer the least bit error rate at the receiver. These performance parameters need to be set right to avoid misplacement of priority in a given communication set up. Knowing the parameters for the different modulation schemes and the relationship between the parameters will enable the designer to make a wise choice on the best suited digital modulation scheme to be used in a particular application. We shall compare different digital modulation schemes based on four different performance parameters - Spectral efficiency, power efficiency and bit error rate and signal-to-noise ratio. We shall calculate their numerical values for a given application, show relationship between them and suggest application where they are best suited for in order to help the designer make a wise choice on the choice of a modulation scheme that best suits his communication link design. As we shall see, we cannot achieve the best value for all four simultaneously; one needs to be traded for another most times. We shall be comparing the Binary phase shift keying (BPSK), Quadrature phase shift keying (QPSK) and 8-Level phase shift keying (8-PSK) modulation schemes.*

**Keywords:** Bandwidth Efficiency, Signal-to-Noise Ratio, Power Efficiency, Bit Error Rate.

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## 1. INTRODUCTION

Our goal is to compare the performance of the digital modulation schemes, BPSK, QPSK and 8-PSK in terms of four performance parameters, Bandwidth Efficiency, Signal to Noise Ratio, Power Efficiency and Bit Error Rate

### Definition of terms:

#### 1.1 Bandwidth efficiency, Spectral efficiency or spectrum efficiency ( $\eta_B$ )

Generally speaking, Efficiency is a quality that characterizes the correspondence between the consumed resources and the attained utility. Bandwidth is a scarce resource and costly one at that and hence we need to utilize the available one efficiently and effectively.

Most wireless transmissions as of today are digital with limited available spectrum; thus the type of modulation employed is crucial. The transition of analogue to digital modulation offered improved data security, enhanced quality communication, additional information-carrying capacity, compatibility with digital data services, swift system availability as well as RF spectrum sharing to accommodate added services [1]. [7].

However, factors such as bandwidth availability, permissible power and inherent noise level of the system are major restrictions developers of communication systems face in the industry which affect spectral efficiency; thus slowing down how fast information can be transmitted in an allotted bandwidth [1].

Bandwidth efficiency refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. It is a measure of the ability of a modulation technique to accommodate data within a limited bandwidth [1]. The bandwidth efficiency of a digital communication system is measured in bit/s/Hz, or less frequently but unambiguously, in (bit/s)/Hz.

Many researchers consider bandwidth efficiency as the principal efficiency criterion [2]. This tendency is vividly reflected by the abundance of standards, such as the High-Speed Packet Access (HSPA) [3]. and the IEEE 802.11n as well as the 802.16 and 3GPP LTE [4]. systems complemented by a large body of theoretical studies proposing transceiver schemes exhibiting an ever-increasing bits - per- second-per-Hertz (bps/Hz) throughput, while sometimes overlooking the associated area spectrum efficiency, power consumption and complexity-related issues.

### 1.2 Power efficiency ( $\eta_p$ )

Power efficiency describes the ability of a modulation technique to preserve the fidelity of a digital message at low power levels. It is the capability of a modulation technique to preserve the bit error probability of the digital message at low power levels [9]. In digital communication, in order to increase noise immunity, it is necessary to increase the signal power [1]. The amount by which the signal power should be increased to obtain a certain level of fidelity (an acceptable signal to noise ratio) depends on the particular type of digital modulation employed. The power efficiency  $\eta_p$  (sometimes called energy efficiency) is measure of how favorably this tradeoff between message fidelity and signal power is made and is often expressed as the ratio of the signal energy per bit to noise spectral density ( $E_b/N_0$ ) required at the receiver input for a certain probability of error.

### 1.3 Signal-to-noise ratio (S/N or SNR)

In the absence of noise, it has been proven by Harry Nyquist [11]. that we can achieve a maximum capacity (maximum information transfer rate or bitrate) that is equal to twice of the bandwidth. That is,

$$C_{MAX} = 2B \text{ ----- (1)}$$

The limitation is due to the effect of intersymbol interference, such as is produced by delay distortion.

If each signal element can represent more than two levels (ie each signal can represent more than one bit as in higher order modulation schemes with multiple bits per symbol) then Nyquist formulation becomes

$$C = 2B \text{ LOG}_2N \text{ ----- (2)}$$

Where, B is the Bandwidth (Hz); N is the Number of coding levels

The above formula, means that we can simply increase bandwidth efficiency (ie increase the information transfer rate at a constant bandwidth) by simply increasing the number of coding levels N. This can be done by using higher number of bits per symbol in modulation. However, we cannot do this without accumulating extra noise (Additive White Guassian Noise, AWGN) into the channel (I.e. increasing the channel's susceptibility to AWGN). The presence of noise can corrupt one or more bits. If the data rate is increased in a given limited bandwidth, then the bits become shorter so that more bits are affected by a given pattern of noise. This limits the maximum capacity we can achieve with a given bandwidth because of the presence of an inherent noise in the channel. All of these concepts can be tied together neatly in a formula developed by the mathematician Claude Shanon. The equation is given below,

$$C = B \text{ log}_2 \left( 1 + \frac{S}{N} \right) \text{ ----- (3)}$$

S/N = Signal to noise ratio in linear scale

Dividing both sides by B, we get the relationship between, the spectral efficiency and the signal to noise ratio

$$\frac{C}{B} = B \text{ log}_2 \left( 1 + \frac{S}{N} \right) = \eta_B \text{ ----- (4)}$$

For a given level of noise, we would expect that greater signal strength would improve the ability to receive data correctly in the presence of the noise. The key parameter involved in this reasoning is the **signal-to-noise** ratio which is the ratio of the power in a signal to the power contained in the noise that is present at a particular point in the transmission [5]. Typically this ratio is measured at the receiver because it is at this point that an attempt is made to process the signal and recover the data. It is either measured in a logarithmic scale as decibel (dB) or in a linear scale.

$$SNR = \frac{\text{signal power}}{\text{noise power}} \text{ ----- (5)}$$

$$SNR_{dB} = 10 \text{ log}_{10} \frac{\text{signal power}}{\text{noise power}} \text{ ----- (6)}$$

Signal to noise ratio is important in the transmission of digital data because it sets the upper bound on the achievable data rate. It should be noted that Shannon formula represents the theoretical maximum that can be achieved. In practice however, much lower data rates are achieved. One reason for this is that the formula assumes only additive white gaussian noise. Other forms of transmission impairments like the impulse noise and path loss are not accounted for.

#### 1.4 Bit Error Rate (BER) or Probability of Error ( $P_e$ )

Bit error rate is the most common measure of error performance in a data circuit and is defined as the probability that a bit received is in error. With other factors held constant, an increase in data rate increases the bit error rate and an increase in SNR decreases the bit error rate [5]. The mathematical formula for bit error rate is given below,

$$P_e = \frac{1}{2} \operatorname{erfc} \left[ \sqrt{\frac{E}{N_o}} \right] \text{-----} (7)$$

Where,  $E/N_o$  is a ratio called Energy per bit to noise spectral density;

$$\left( \frac{E_b}{N_o} \right)_{dB} = (SNR)_{dB} - \left( \frac{C}{B} \right)_{dB} \text{-----} (8)$$

erfc is an error function used in estimating probability of error. This easily can be estimated using the MATLAB command 'erfc(x)'

#### 1.5 Relationship between Bandwidth Efficiency, Signal-to-Noise Ratio, Power Efficiency and Bit Error Rate

Higher order modulation schemes (schemes that employ higher number of bits per symbol) are able to achieve higher spectral efficiencies at the expense of increased noise susceptibility of the channel. This leads to low SNR and consequently high BER. Therefore such higher order modulation schemes need to have high signal to noise ratio in order to be able to preserve the fidelity of the information transmitted. Increasing the signal to noise ratio can be achieved by increasing the transmission power of the signal so that the signal power becomes significantly higher than noise power. However, increasing the signal power reduces the power efficiency and makes the signals more susceptible to interference especially in cellular networks. Therefore a tradeoff between these three parameters is usually reached to determine the modulation scheme best suited for a given application.

For example, a host of spread-spectrum methods intentionally sacrifice the bps/Hz performance (i.e. the bandwidth efficiency) for the sake of achieving a better bit-per-second-per-Watt (i.e. the power efficiency), typically accompanied by lower levels of spectrum contamination, as well as by an increased robustness against the interference [6].[8]. By no means should these methods be classified as less efficient, since in the appropriate circumstance/application they are capable of considerably improving the overall performance of the entire network.

That is to say that, to increase capacity, we just need to increase the number of bits per symbol (achieved by using higher order modulation schemes). However, increasing the modulation efficiency leads to higher susceptibility of the channel to noise. Hence to overcome this noise to a reasonable extent and consequently decrease the bit error rate we need to increase the signal to noise ratio by increasing the signal power. Unfortunately for us increasing the signal power is also bad for us as it decreases our power efficiency and causes more tendency of interference especially in cellular network applications.

Other factors that also influence the choice of digital modulation scheme in a given application are effect of multipath and fading condition on the modulated signal and the cost of deployment of such scheme [1].

As presence of noise and other channel impairments limits the maximum capacity that can be achieved with a given a given bandwidth, so are there other factors which also influences the capacity that can be achieved with a given bandwidth, such factors includes,

1. The number of sidebands in its frequency spectrum. Use of single sideband modulation increases the bandwidth efficiency by double as compared to what can be achieved using double sideband modulation techniques. [10].
2. The line code used - Return to zero (RZ) signaling techniques has been known to occupy twice the bandwidth as the non return to (NRZ). Hence The NRZ signal can achieve twice as much spectral efficiency as the RZ signals
3. Use of effective and efficient multichannel access techniques can greatly increase the spectral efficiency of that modulation scheme
4. The spectral shaping of the transmitted pulses also influences the value of the spectral efficiency. Smoothed pulse waveform has been known to achieve higher spectral efficiencies than the unsmoothed pulse waveforms.

## 2. BINARY PHASE SHIFT KEYING (BPSK)

In BPSK, the phase of constant amplitude carrier signal is switched between two values to represent the two binary digits (i.e. 0 and 1). One phase is used to represent a binary one while another phase represents a binary 0. The phases are separated by 180° or π radians

The modulated BPSK signal is given by

$$S_{BPSK}(t) = m(t) \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \text{----- (9)}$$

Where, m(t) = binary bit (1 for binary and -1 for binary 0); f<sub>c</sub> = frequency of the the carrier;

θ<sub>c</sub> = phase of the carrier; E<sub>b</sub> = Energy per bit which is related to the amplitude of the carrier by,

$$E_b = \frac{1}{2} A_c^2 T_b \text{----- (10)}$$

Where, A<sub>c</sub> = amplitude of the carrier signal; T<sub>b</sub> = duration of one bit (in sec)

Now, let us assume that we are allocated with 1MHz bandwidth in a channel with SNR of 12dB. Let's show how we can determine the other performance parameters and how they are related to each other.

From Equation (3),  $C = B \log_2 \left( 1 + \frac{S}{N} \right)$

Converting 12dB to a linear scale,  $10 \log_{10} W = 12$  hence  $W = 10^{1.2} = 15.85$

So, SNR = 15.85,  $B = 1 \times 10^6 = 10^6$

$C = 10^6 \times \log_2 (1 + 15.85) = 4074676 \approx 4.1 \times 10^6 = 4.1 \text{Mbps}$

Bandwidth efficiency,  $\eta_B = \frac{C}{B} = \frac{4.1 \text{Mbps}}{1 \text{Mhz}} = 4.1 \text{bps/Hz}$

Bandwidth efficiency in dB, =>  $10 \log_{10} 4.1 = 6.13 \text{dB}$

Applying equation (8),  $\left( \frac{E_b}{N_o} \right)_{dB} = (SNR)_{dB} - \left( \frac{C}{B} \right)_{dB}$ ; =>  $\left( \frac{E_b}{N_o} \right)_{dB} = 12 \text{dB} - 6.13 \text{dB} = 5.87 \text{dB}$

$E_b/N_o = 5.87 \text{dB}$  converting to linear scale =  $10^{0.587} = 3.864$

Finding the bit error rate,  $P_e = \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{E}{N_o}} \right] = \frac{1}{2} \text{erfc} [\sqrt{3.864}] = 0.5 \times \text{erfc} [1.966]$

In MATLAB command window, type, 'erfc(1.966)' then press enter,

erfc(1.966) = 0.0054

hence  $P_e = 0.5 \times 0.0054 = 0.0027$

Therefore, the value BER is 0.0027

### 2.1 Quadrature Phase Shift Keying (QPSK)

In QPSK, the phase of a constant amplitude carrier signal is switched between four values to represent four binary values (i.e. 00, 01, 10 and 11). Each of these binary values is called symbols. Because QPSK allows us to use two binary digits (instead of one as in BPSK) to represent a symbol which corresponds to given phase value of the carrier, hence it can achieve twice the bandwidth efficiency as BPSK. One phase is used to represent the symbol 00 while another phase represents the symbol 01 and so on. The phases are separated by 90° or π/2 radians

If we still assume the same bandwidth and SNR here, we can try to calculate the value of BER to see the influence of the higher bits per symbols employed in QPSK on its bit error performance.

$B = 1 \text{MHz} = 10^6 \text{Hz}$ ; SNR = 12dB

Since QPSK uses 2bits per symbol, the channel capacity/data rate (C) and bandwidth efficiency (η<sub>B</sub>) will be twice of that of BPSK.

Hence,  $C = 8.2 \text{Mbps}$  and Bandwidth efficiency,  $\eta_B = \frac{C}{B} = \frac{8.2 \text{Mbps}}{1 \text{Mhz}} = 8.2 \text{bps/Hz}$

Bandwidth efficiency in dB, =>  $10 \log_{10} 8.2 = 9.14 \text{dB}$

Applying equation (8),  $\left( \frac{E_b}{N_o} \right)_{dB} = (SNR)_{dB} - \left( \frac{C}{B} \right)_{dB}$ ; =>  $\left( \frac{E_b}{N_o} \right)_{dB} = 12 \text{dB} - 9.14 \text{dB} = 2.86 \text{dB}$

$E_b/N_o = 2.86 \text{dB}$  converting to linear scale =  $10^{0.286} = 1.932$

Finding the bit error rate,  $P_e = \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{E}{N_o}} \right] = \frac{1}{2} \text{erfc} [\sqrt{1.932}] = 0.5 \times \text{erfc} [1.39]$

In MATLAB command window, type, 'erfc(1.39)' then press enter,

erfc(1.39) = 0.0493

Hence  $P_e = 0.5 \times 0.0493 = 0.02465$

Therefore, the value of BER for the QPSK is 0.02465

### 8-PSK

In 8-PSK, the phase of the carrier signal is switched between eight values to represent eight binary symbols (i.e. 000, 001,.....,111). Because QPSK allows us to use three binary digits (instead of one as in BPSK and two in QPSK) to represent a symbol it can therefore achieve trice the bandwidth efficiency as BPSK. One phase is used to represent the symbol 000 while another phase represents the symbol 001 and so on up to 000. The phases are separated by 45° or π/4 radians

Let's consider,

$$B = 1\text{MHz} = 10^6\text{Hz}; \text{SNR} = 12\text{dB (As before)}$$

Since QPSK uses 3bits per symbol, the channel capacity/data rate (C) and bandwidth efficiency (η<sub>B</sub>) will be trice of that of BPSK.

$$\text{Hence, } C = 12.3\text{Mbps and Bandwidth efficiency, } \eta_B = \frac{C}{B} = \frac{12.3\text{Mbps}}{1\text{MHz}} = 12.3\text{bps/Hz}$$

Bandwidth efficiency in dB, => 10log<sub>10</sub> 12.3 = 10.9dB

$$\left(\frac{E_b}{N_o}\right)_{dB} = (\text{SNR})_{dB} - \left(\frac{C}{B}\right)_{dB}; \Rightarrow \left(\frac{E_b}{N_o}\right)_{dB} = 12\text{dB} - 10.9\text{dB} = 1.1\text{dB}$$

$$E_b/N_o = 1.1\text{dB converting to linear scale} = 10^{0.11} = 1.288$$

$$\text{Finding the bit error rate, } P_e = \frac{1}{2} \text{erfc} \left[ \sqrt{\frac{E}{N_o}} \right] = \frac{1}{2} \text{erfc} [\sqrt{1.288}] = 0.5 \times \text{erfc} [1.135]$$

In MATLAB command window, type, 'erfc(1.39)' then press 'Enter',

$$\text{erfc}(1.135) = 0.1085$$

$$\text{Hence } P_e = 0.5 \times 0.1085 = 0.05425$$

Therefore, the value of BER for the 8 PSK is 0.05425

The values of η<sub>B</sub>, η<sub>P</sub>, P<sub>e</sub> at 1MHz bandwidth and SNR of 12dB are shown in the table below

**Table 1.1. Values of η<sub>B</sub>, η<sub>P</sub>, P<sub>e</sub> at 1MHz bandwidth and SNR of 12dB**

	<b>BPSK (1bit/symbol)</b>	<b>QPSK (2bits/symbol)</b>	<b>8-PSK (3bits/sybol)</b>
<b>Bandwidth Efficiency (η<sub>B</sub>)[bps/Hz]</b>	4.1	8.2	12.3
<b>RelativePower Efficiency (η<sub>P</sub>)</b>	3.864	1.932	1.288
Bit error rate (P <sub>e</sub> )	0.0027	0.02465	0.05425

### 3. RECOMMENDATION AND CONCLUSIONS

As we can see from our comparison that BER increases for higher order modulation schemes (using more bits per symbol). This means there is more likelihood of bit errors in such schemes. Therefore to overcome this problem, we need to increase the energy per bit to noise spectral density ratio and this can be achieved by increasing the signal power in order to force the signal to noise ratio to increase. The signal power is increased to meet a specified BER which we desire for the optimum performance of that communication system. We cannot just increase power indiscriminately because doing that will also put a strain on the desired power efficiency.

Therefore, if in the application, achieving the maximum data rate possible with the available limited bandwidth is the primary focus, then higher order modulation schemes should be employed. But however if using the minimal power to achieve the maximum throughput is the is the primary concern, then lower order modulation schemes should be considered.

It also worthy to note that use of higher order modulation schemes will require us to use more sophisticated error correction codes. These error correction codes always occupy a reasonable amount of the available bandwidth and thereby reducing the useful throughput and thus the spectral efficiency achievable. More so, higher order modulation schemes require more complex receiver circuit in order to handle the high rate of the incoming data.

For this reason, BPSK should be applied in low power, low bitrate and low cost applications like the Radio frequency identification (RFID) standards such as ISO/IEC 14443. It is presently the predominant standard for biometric passports, credit cards such as the American Express's ExpressPay

QPSK is still widely used in the streaming of SD satellite channels and some HD channels High definition programming is delivered almost exclusively in 8-PSK due to the higher bitrates of HD (high definition) video and the cost of satellite bandwidth. [10].

Higher order modulations are also available in the phase shift keying. For example the 16-psk which uses 4bits/symbol finds wild application in satellite communication

In 16-PSK, following our normal method, for B = 1MHz channel and SNR = 16dB

Then, C = 16.4Mbps and  $\eta_B = \frac{C}{B} = 16.4$

Bandwidth efficiency in dB,  $\Rightarrow 10\log_{10} 16.4 = 12.15\text{dB}$

$$\left(\frac{E_b}{N_o}\right)_{dB} = (SNR)_{dB} - \left(\frac{C}{B}\right)_{dB}; \Rightarrow \left(\frac{E_b}{N_o}\right)_{dB} = 12\text{dB} - 12.15\text{dB} = -1.15\text{dB}$$

The negative sign above suggests that noise spectral density has exceeded energy per bit and this will make it impossible to detect such signal at the receiver. This is because the number of bits per symbol was increased without a corresponding increase in the signal power needed to achieve a desirable signal to noise ratio which is necessary to reliably transmit the signal. It is therefore impracticable to transmit this signal at such modulation level and signal to noise ratio

Hence, the designer needs to take note of these factors in making decision on the choice of digital modulation scheme to use in designing his communication link so that he doesn't trade off more important performance metrics to the less important ones.

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