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Automated Test Station for Low-Pass Filter Cut-off Frequency Measurements

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ABSTRACT

The primary focus of this research paper is to study the automated test station to determine the cut-off frequency. The study was conducted for the low-pass filter in the frequency range 50 Hz – 100 KHz. Preliminary test was conducted using 10 filters whose cut-off frequencies were measured. The main blocks of the station are described, emphasizing some originality. The test results depict high degree of accuracy as such were manually checked to confirm the effectiveness of the test station. The automated test station was also developed to include provisions which allow the results to be stored in a spreadsheet file against each of the filter serial number. This is a unique feature of the measurement system.

***Keywords*:** Automation; Low-pass filter; Cut-off frequency; Measurement; Signal filtering.

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1. INTRODUCTION

Electronic filters are electronic circuits which attenuate or remove unwanted frequency components from the signal to enhance wanted ones. Electronic filters can be active or passive, digital or analog, high-pass, low-pass, band reject or all pass, discrete or continuous filters [1].

A low-pass filter is a filter that passes low-frequency signals but attenuates (reduces the amplitude of) signals with frequencies higher than the cut-off frequency f_c. An ideal low-pass filter completely eliminates all frequencies above the cut-off frequency f_c (called the reject band) while passing those below (the pass band) unchanged [1, 2]. The frequency response of an ideal low pass filter is a rectangular function and is depicted in Figure 1.

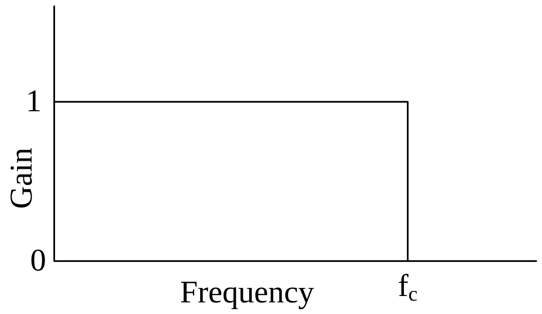

Figure 1: Amplitude response for ideal low-pass filter

Ideal filter is not physically realizable, a more realizable filter (real filter) is implemented using a simple RC circuit. A real low-pass filter has a longer roll off of -20db/decade and the amplitude (gain) defines the attenuation of the input signal as it passes through the low-pass filter [1, 3]. A real low-pass fiter characteristics is depicted in Figure 2.

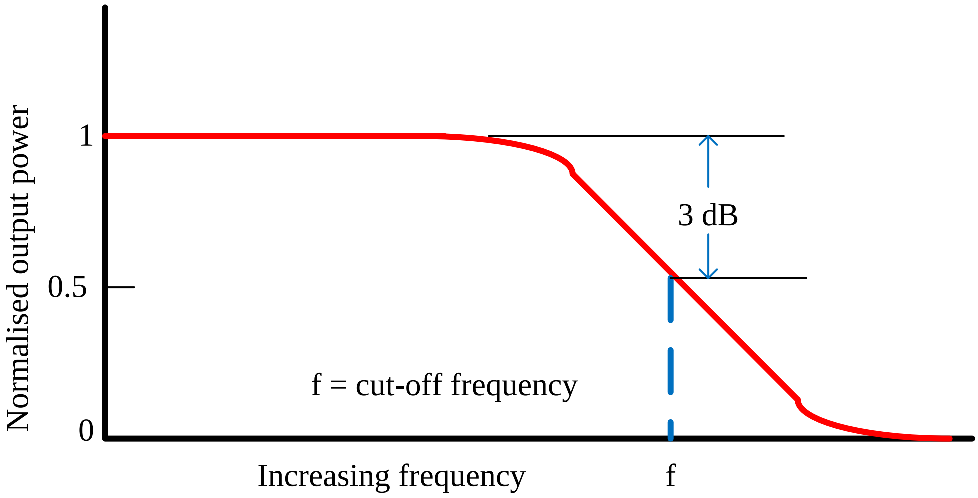

Figure 2: A real low-pass filter characteristics

The 3db or the cut-off frequency point is the point where the amplitude drops to $\frac{1}{\sqrt{2}}$ of the output in the flat portion of the filter response. Practical implementation of a low-pass filter is governed by the simple RC circuit depicted in Figure 3.

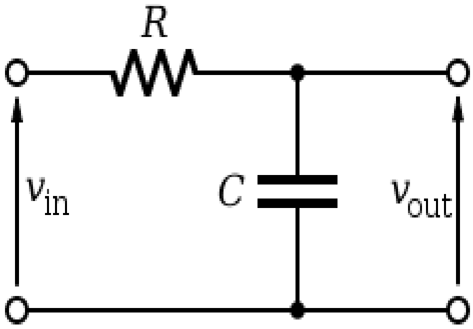

Figure 3: Simple RC low pass filter

By the principle of voltage divider rule, the mathematical relationship between the input V_{in} and V_{out} can be written as:

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{jWC}}{R + \frac{1}{jWC}} \tag{1}$$

The laplace description of damped sinusoid is thus;

$$s = \sigma + jw \tag{2}$$

In Equation (2), the real part σ is the attenuation of a sinusoid describing that part of a damped sinusoid and w is the input frequency associated with this complex number or signal. For a pure sinusoid, the real part of Equation (2) is zero and therefore Equation (2) approximately becomes:

$$s = jw \tag{3}$$

Substituting Equation (3) in Equation (1) yields

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{SC}}{R + \frac{1}{SC}} \tag{4}$$

Transforming Equation (4) yields

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$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{RC}}{S + \frac{1}{RC}} \tag{5}$$

From Equation (1) and Equation (4), the magnitude can be written as:

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\frac{1}{RC}}{\sqrt{\left(\frac{1}{RC}\right)^2 + w^2}} \tag{6}$$

From Equation (6), it could be seen that, if w = 0, the magnitude is = 1 and if w gets larger and larger i.e $w \rightarrow \infty$, the magnitude → 0. On the other hand, if $w = \frac{1}{RC}$, then the magnitude $\left|\frac{V_{out}}{V_{in}}\right| \approx \frac{1}{\sqrt{2}} \approx 0.7071$. Thus, the cut-off frequency for the simple RC circuit is $\approx \frac{1}{RC}$, $\therefore w_c = \frac{1}{RC}$ and thus $2\pi f_c = \frac{1}{RC}$.

$$f_c = \frac{1}{2\pi RC} \tag{7}$$

Therefore Equation (7) is called the cut-off frequency of RC low-pass filter. Taking the 20log of ($\frac{1}{\sqrt{2}}$ = 07071) i.e 20log (0.7071) = - 3dB. This implies that the cut-off frequency has been defined interms of the -3dB point [1-4].

In electrical/electronics applications, the design efficiency of filters to meet the desired specification and accuracy is determined by checking the filter frequency response through validation of the designed test measurement station. The accuracy in determining a filter cut-off frequency ensures that unwanted signal are perfectly removed from the signal of interest without attenuation. In communication systems where noise cancellations are required particularly in modems, filter frequency response is vital due to the closeness of the noise signal to the intelligent signal. Notwithstanding, the importance of filtering in signal processing for improved efficiency and performance, this work is aimed to develop an automated test station for the measurement of low-pass filter (LPF) cut-off frequency that are commonly used in audio applications [4].

Automation on the other hand, is the shift from manual or paper based ways of working to a new automatic software based or data driven practices. This enables applications to produce by-products in a sustainable passion. In this paper, the basic structure of the automated measurement system consist of the controller and measurement instruments that communicates through the GPIB, RS232 and RS432 interfaces. The developed automated test station for determining the cut-off frequency of the LPF will have the advantages of high speed measurement, repeatability, high accuracy and non speciality requirement for the user [5].

However, in recent years there has been several studies in the literature reporting automated test measurement systems [6-9].

2. METHODS OF DETERMINING THE LPF CUT-OFF FREQUENCY

Different techniques exist that could be used to estimate the cut-off frequency of a low-pass filter, these are outlined in this section.

I. Frequency Response Technique

For the low-pass filter, V_{out} is always less than or equal to V_{in}, this is usually a fraction which is less than 1, so the log of this function will be zero or negative. However regardless of the filter component values, there will be at least one particular frequency at which $R = X_c$ (for RC filter) or $R = X_L$ (for RL filter). At this frequency, $\frac{V_{out}}{V_{in}} = 0.7071$ and the gain or attenuation = 20log (0.7071) = - 3dB [1 – 3]. This point is clearly shown on the graph in Figure 4.

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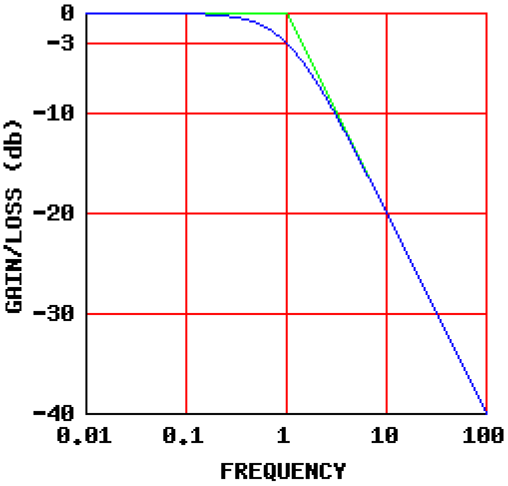

Figure 4: LPF frequency response plot

At lower frequency, the attenuation is reduced and rapidly becomes 0 dB, and at higher frequency attenuation increases rapidly until it falls off at a constant slope of 20 dB per decade as shown in Figure 4.

II. Phase Response Technique: For LPFs V_{out} always lags V_{in} by some phase angle between 0^o and 90^o. Most of the variation occurs within one decade of the cut-off frequency as shown in Figure 5.

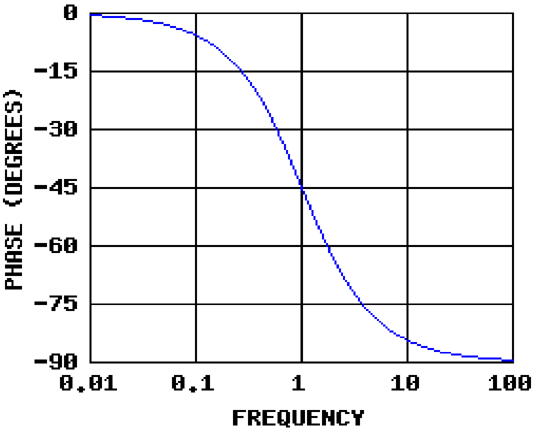

Figure 5: Phase response plot

At frequencies well below the cut-off frequency f_c, there is essentially no phase shift. The phase lag begins to become more significant about a decade below f_c and reaches 45^o at f_c itself. Above the cut-off frequency f_c, the output phase continuous to change rapidly during the first decade, by that time the phase lag is close to 90^o [2].

III. Output Voltage to Input Voltage Ratio Technique: This is the technique employed in this paper. In this technique, the relationship between the output and input voltages at the cut-off frequency is used. At cut-off frequency, the ratio $\frac{V_{out}}{V_{in}}$ = 0.7071. The gain of the LPF changes at different frequencies and comparison is being made with the ratio $\frac{V_{out}}{V_{in}}$ = 0.7071 at cut-off frequency [1-3].

IV. Output Power to Input Power Ratio Technique: This technique is similar to the technique of the output voltage to input voltage ratio technique. In this case, the power is proportional to the square of the voltage i.e $P \alpha V^2$ with the constant of proportionality being the reciprocal of Z. Thus, the gain is defined as: $10\log\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{V_{out}}{V_{in}}\right)^2 = 10\log\left(\frac{1}{\sqrt{2}}\right)^2$ at cutoff frequency. This effectively means that the output power to input power ratio is 0.5 at the cut-off frequency [1].

V. Time Constant Technique: In this technique, Equation (8) is considered with a step input to be applied to the filter.

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$$V_o(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right) \tag{8}$$

In Equation (8) V_s is the input voltage and τ = time constant. Assuming a step input of magnitude A is applied to Equation (8) and transforming gives $V_o(t) = \left(A - Ae^{-\frac{t}{\tau}}\right) = \frac{V_o(t)-A}{-A} = \left(e^{-\frac{t}{\tau}}\right)$. Taking the natural log of both side yields;

$$In\left[\frac{V_o(t) - A}{-A}\right] = -\frac{t}{\tau} \tag{9}$$

Plotting $In\left[\frac{V_o(t)- A}{-A}\right]$ against t gives a linear relationship with its gradient as the negative of 1 upon τ. The negative value indicates that the slope is a decreasing slope [1]. The cut-off frequency can then be deduced as in Equation (10)

$$f_c = \frac{1}{2\pi\tau} \tag{10}$$

3. MEASUREMENT SETUP

I. Hardware Setup

The hardware components include function generator, oscilloscope, NI-PXI-1042 with E series data acquisition card, GPIB instruments, digital multi-meter, low-pass filter, connecting cables and PC monitor. The PC is fitted with the IEEE interface. These components are connected as shown in the block diagram in Figure 6.

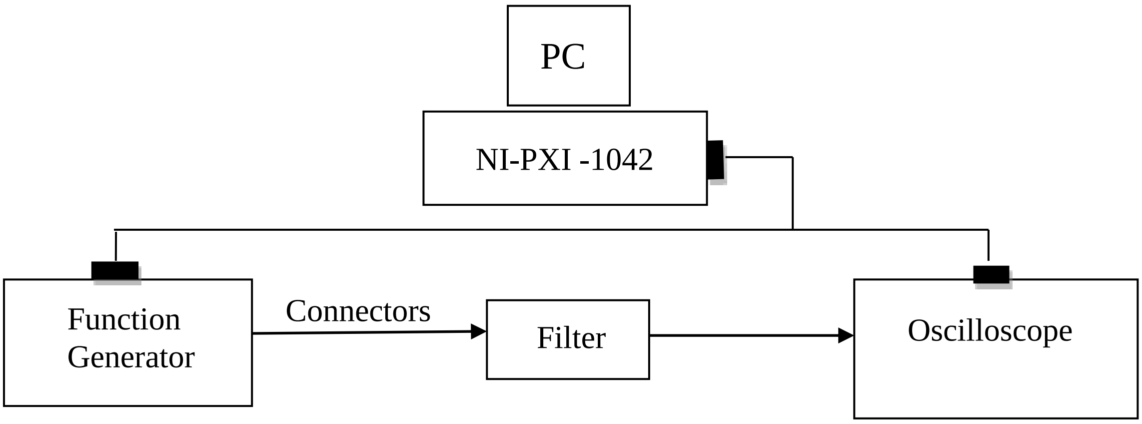

Figure 6: Hardware setup

II. Software Description

The LabView program was used to realize the automated measurement task for the determination of the low-pass filter cut-off frequency. The hardware setup of Figure 6 is simulated using the labView VI's. The input of the VI is the frequency and the voltage settings for the function generator and the output is the voltage measured.

The ***for*** loop was programmed to execute 14 iterations to cover the frequency range (50 Hz – 10 KHz). During each iteration, the loop iteration value is manipulated with a constant number of 14 to generate the frequency using the appropriate VI's. This frequency combined with the frequency stored in a file which has a default value of 0 Hz, serve as the input frequency.

The generated frequency is converted into string data using the convert number to decimal string VI, suitable for the **GPIB** write VI which writes the data to the function generator as identified by the address string. The address string then takes the address of the device. Similarly, a voltage of magnitude 10 VPP was set using the **GPIB** write VI which writes the value to the function generator using the standard command. The function generator is always set on output load impedance high in order to match the high input impedance of the oscilloscope to avoid loading effect.

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The 2 number **GPIB** write VI's of the oscilloscope identified by its address tell the oscilloscope to write the value of the input and output voltages on the appropriate channel using the command. Similarly, the 2 number **GPIB** Read VI's of the oscilloscope read the input and output voltages which are converted to numbers using convert fractional/exponential string to number VI. The oscilloscope is always set on auto-scale using the command in order to evaluate all input waveforms and find the optimum conditions for displaying the waveform. These are then fed into divide VI which divides the output voltage by the input voltage. Since the technique used is the $\frac{V_{out}}{V_{in}} = 0.7071$ at cut-off, the ratio $\frac{V_{out}}{V_{in}}$ is then compared with the cut-off frequency voltage ratio value of 0.7071 using the greater comparison VI. If the $\frac{V_{out}}{V_{in}}$ ratio is greater than 0.7071(i.e. the condition is true), then this allows the true condition of the case structure to be executed by writing (storing) the frequency in the file, otherwise nothing will be stored. The process continues the final iteration has been reached and the value of the frequency at the final iteration corresponds to the approximate value of the cut-off frequency and this value will be displayed on the front panel.

Finally, another ***for*** loop set to iterate 10 times was included in the program to enable the results from the 10 filters to be written in the spreadsheet file. This was achieved by including prompt user VI in the loop which prompts the user to input the filter number during each of the 10 iterations, and the filter number and the cut-off frequency appended in the file.

4. RESULTS AND EVALUATION OF PERFORMANCE

This section presents the results thus obtained. Using the hardware setup of Figure 6 and without the filter connected, a voltage of magnitude 10V peak-to peak (10Vpp) was set on the function generator which was set on high output load impedance and a corresponding measurement of the input and output voltage on the oscilloscope set auto-scale were both 10.6Vpp which is not exactly the same as the voltage from the function generator. This is due to the limitation placed on the instrument design, by not getting the exact value instead almost close.

Now, with the LPF filter connected and the voltage set at 10Vpp, its input and output voltages were obtained by varying the frequency on the function generator. The corresponding ratio of the output to the input voltage was obtained. Since the approach used is the $\frac{V_{out}}{V_{in}} = 0.7071\ (-3dB)$, the ratio obtained is compared with this value and if it is equal or close to, and this gives an indication of where the cut-off frequency lies. Different ratios were obtained and recorded by varying the frequency, and the graph of gain (dB) against the frequency (Hz) was plotted. The low-pass filter 17 (i.e LPF 17) was considered in the analysis. The results were tabulated as shown in Table 1.

Table 1: Shows the value of input and output voltages obtained by varying the frequency, the gain by dividing the output by the input and taking the 20log of the result. This result is for filter 17 i.e LPF 17.

Table 1: The value of input and output voltages obtained by varying the frequency, the gain

Frequency (Hz)	V_{in} (V)	V_{out} (V)	V_{out}/V_{in}	Gain (dB)
10	10.6	10.3	0.9717	-0.2494
20	10.6	10	0.9434	-0.5061
30	10.6	9.7	0.9151	-0.7707
40	10.6	9.3	0.8774	-1.1365
50	10.6	8.8	0.8302	-1.6165
60	10.6	8.3	0.7830	-2.1246
70	10.6	7.8	0.7358	-2.6642
80	10.5	7.3	0.6952	-3.1573
90	10.5	7.0	0.6667	-3.5218
100	10.5	6.7	0.6381	-3.9023

Plotting the values of the gain (dB) against the frequency (Hz) yields the graph depicted in Figure 7.

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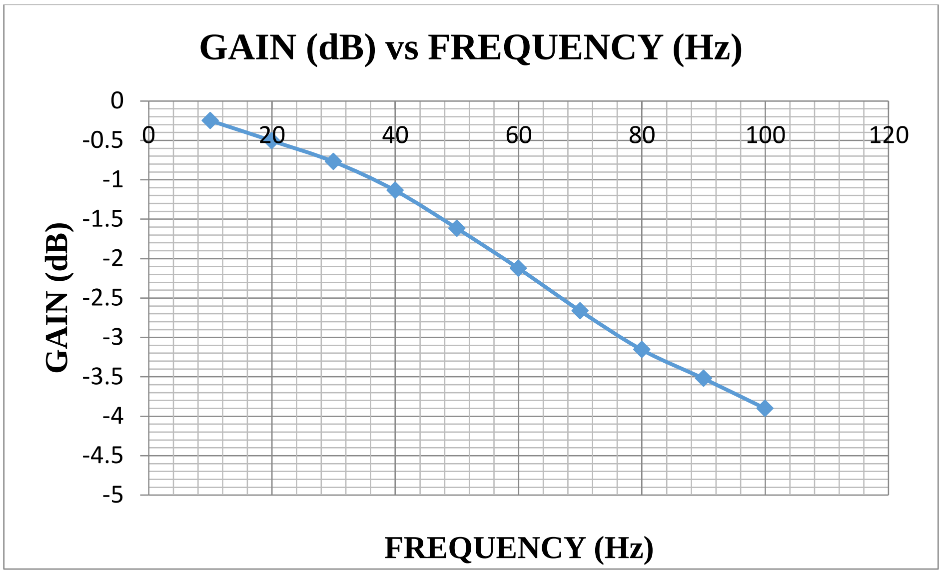

Figure 7: The plot of the gain (dB) vs Frequency (Hz) for filter 17 (LPF 17)

From Figure 7, it can be seen clearly that at -3dB point the corresponding frequency is about 78 Hz. When the labView program was run ten times, six of the readings obtained were 78 Hz, which means that the precision of the system is fairly good but that does not guarantee accuracy of the automated test station. The uncertainty of the system results will be analyzed. The same procedure was applied to the other 9 filters.

The results of the automated test station from measuring the cut-off frequencies of the 10 filters was tabulated as shown in Table 2.

Table 2: Shows the cut-off frequencies of the 10 filters when the labView program of the automated test and measurement system was run ten times.

Table 2: The Cut-off frequencies of the 10 filters

			Cut-off Frequency (Hz)							
Filter No.	**f_{c1}**	**f_{c2}**	**f_{c3}**	**f_{c4}**	**f_{c5}**	**f_{c6}**	**f_{c7}**	**fc_8**	**fc_9**	**fc_{10}**
LPF 17	74	78	78	78	82	82	74	78	78	78
LPF 09	206	202	206	206	206	206	210	210	202	206
LPF 07	250	250	246	246	254	250	250	250	250	254
LPF 16	98	94	94	94	94	90	94	90	98	94
LPF 04	378	378	382	378	378	378	374	378	374	378
LPF 24	954	950	946	950	950	950	950	950	946	950
LPF 22	1110	1110	1110	1110	1110	1114	1106	1106	1110	1110
LPF 11	8332	8336	8332	8336	8336	8336	8336	8332	8336	8336
LPF 03	1338	1338	1338	1338	1334	1334	1338	1334	1334	1338
LPF 15	1548	1548	1552	1552	1552	1552	1552	1552	1552	1552

Table 2 shows the results obtained from the 10 different filters when the labView program of the automated test station was run ten times for each of the LPFs. To analyze for the uncertainty in the measurement, type A approach will be considered. This is because it is based on statistical analysis and there were some variations in the readings obtained i.e none of the filters have repeated measurements throughout when the labView program was run. This will at least give information about the spread of uncertainty in all the measurements taken. However, one of the filters will be considered and the result obtained will be used to generalize the uncertainty in the system. For LPF 17, the type A analysis steps is as shown by first considering Table 3.

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Table 3 shows the cut-off frequencies for filter 17 by running the program ten times, difference of each of the readings and the mean and the square of the difference.

Table 3:The cut-off frequencies for filter 17

S/N	Cut-off Frequency (Hz)	d = x-µ	d*d
1	74	-4	16
2	78	0	0
3	78	0	0
4	78	0	0
5	82	4	16
6	82	4	16
7	74	-4	16
8	78	0	0
9	78	0	0
10	78	0	0
Total	780	0	64

The best estimate or mean of these results can be computed using the equation given by:

$$\mu = \frac{\sum_1^n \mathrm{x_i}}{n}$$

$$\mu = \frac{780}{10}$$

$$= 78\,Hz$$

Also, from Table 3 the sample standard deviation, the population standard deviation and the standard uncertainty were calculated as shown thus;

Sample Standard deviation,

$$\sigma_{\mathrm{n}} = \sqrt{}[\frac{\sum_1^n \mathrm{d}^2}{n}]$$

$$\sigma_{\mathrm{n}} = \sqrt{(64/10)} = 2.5298$$

The population standard deviation,

$$\sigma_{\mathrm{n-1}} = \sqrt{}[\frac{\sum_1^n \mathrm{d}^2}{n-1}]$$

$$\sigma_{\mathrm{n-1}} = \sqrt{64}/9) = 2.6667$$

⇒ the type A standard uncertainty,

$$s_n = \frac{\sigma_{\mathrm{n-1}}}{\sqrt{n}}$$

$$s_n = \frac{2.6667}{\sqrt{10}} = 0.8433$$

Since ten measurements were taken, the number of degrees of freedom is 9 (i.e one less than the number of measurements taken). To obtain 95% probability range in the measurement, the student's t distribution table would be considered. From the table t_{95} corresponding to 9 degrees of freedom is 2.26.

∴ 95% confidence limits for the uncertainty in the estimate are:

± 2.26 x 0.8433

± 1.9058

The result would therefore be quoted as 78 Hz ± 1.9058 Hz. This means that the true value of the result is 95% probable to lie in that range. The best estimate for the cut-off frequency of the other filters was obtained and expressed in the 95% confidence limit for the uncertainty in the estimate (generalizing the uncertainty value to the other filters) is as shown in Table 4.

Table 4: Shows the best estimate of the cut-off frequency for each of the filter and the 95% confidence limit in the estimates.

Table 4: Shows the best estimate of the cut-off frequency for each of the filter

Filter Number	**Cut-off Frequency (Hz)**
LPF 17	78 ± 1.9058
LPF 09	206 ± 1.9058
LPF 07	250 ± 1.9058
LPF 16	94 ± 1.9058
LPF 04	378 ± 1.9058
LPF 24	950 ± 1.9058
LPF 19	1110 ± 1.9058
LPF 14	8335 ± 1.9058
LPF 03	1336 ± 1.9058
LPF 15	1551 ± 1.9058

4.1 Resolution and Time Response of the System

The system software was designed based on successive approximation approach, and therefore the loop was set to iterate 14 times to cover the frequency range (50 Hz - 10 KHz). Consequently, the system was designed to generate frequency starting 2^{14} – 2^1. Therefore 2^1 Hz is the smallest input frequency to the system. And therefore the system resolution is 2 Hz.

The loop was delayed for 0.2 seconds; therefore time to complete measurement will be 0.2 x 14 = 2.8 seconds.

5. CONCLUSION

The automated test station was developed using LabView program and was used to measure the cut-off frequencies of the 10 filters and the results obtained have been presented as shown in Table 2. Furthermore, it was observed that, there were some kind of uncertainty in the measurement results so obtained and this led to the analysis of uncertainty in the measurement results.

However, out of the ten filters whose cut-off frequencies were measured, it was observed that some gave closely repeated or precise readings with some slight variations in one or two occasions. And this does not guarantee that the results were accurate. Since there were some variations in the measurements obtained, the type A uncertainty analysis was used to analyze the uncertainty in the measurements. Moreover, because the true value of the filters' cut-off frequency was not known which the basis of this assessment task was, the best estimate for it was obtained by averaging the results for each of the filter. And by using the student's t distribution table, the 95% confidence limits in the measurement was estimated. This was used together with the best estimate for each of the filter to establish the fact that the true value is 95% probable to lie in the range quoted for each of the filter as presented in Table 4.

Furthermore, the automated test station was developed to include provisions which allow the results to be stored in a spreadsheet file against each of the filter serial number. This was achieved by prompting the user, using the prompt user VI to input the filter serial number at each time the measurement is to being made and the result appended in the file.

Finally, it could be deduced that, within the limits of experimental errors the objectives of this assessment task were achieved.

REFERENCES

1. Bentley, J. P. 2005. Principles of Measurement Systems, Fourth Edition, Prentice Hall, USA. ISBN: 978 – 0 – 13043028-1.
2. Ibrahim, O., and Akorede, M. F. 2016. Development of Automated Test Measurement System for Determining Low Pass Filter Cut-off Frequency. Acta Technica Corviniensis, Vol. IX (3), Pp. 113 – 117.
3. Harowitz, P., Hill, W., and Hayes, T. C. 1989. The Art of Electronics. Vol. 2, Cambridge University Press, Cambridge.
4. Wong, J. 2009. Broadband Active Filters Solve Signal Processing Accuracy and Matching Problems.
5. Dustin, E., Garrett, T., and Gauf, B. 2009. Implementing Automated Software Testing: How to Save Time and Lower Costs While Raising Quality; Pearson Education.

DOI: 10.31695/IJASRE.2022.8.9.3

6. Melninkaitis, A., Miksys, D., Balciunas, T., Balachninaite, O., Rakickas, T., Grigonis, R., and Sirutkaitis, V. 2008. Automated Test Station for Laser Induced Image Threshold Measurements According to ISO 11254 – 2 Standard. *In Proceedings of SPIE, Vol. 8101 (81011J).*
7. Stratan, A., Zorilla, A., Rusen, L., Simion, S., Blanaru, C., Fenio, C., Neogu, L and Nemes, G. 2012. Automated Test Station for Lase- Induced Damage Threshold Measurements According to ISO 21254 – 1, 2, 3, 4 Standards. *In Proceedings of SPIE, Vol. 8530 (85301Y-1).*
8. Payri, F., Olmeda, D., Guardiola, C., and Martin, J. 2011. Adaptive Determination of Cut-off Frequencies for Filtering the In – cylinder Pressure in Diesel Engine Combustion Analysis. Applied Thermal Engineering, Elsevier, Vol. 31 (14-15), Pp. 2869.
9. Kwak, M. H., Han, K., Kang, Y., Ahn, D., Suh, J. S., and Kim, S. H. 2001. Design of High Temperature Superconducting Low Pass Filter for Broad Band Harmonic Rejection. Applied Superconducting IEEE Transactions, Vol. 11, Pp. 4023 – 4026.

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