

Modelling and Simulation of Acceleration Model of Small Electric Motor Vehicle

Ramesh G Pungle

Associate Professor

Department of Mechanical and Automation Engineering

P. E. S. College of Engineering

Aurangabad, Maharashtra

India

ABSTRACT

This paper presents the development of a mathematical model for tractive effort and acceleration of a small electric motor vehicle/car which is propelled by a permanent magnet direct current (PMDC) motor. The proposed vehicle acceleration model is obtained from torque-speed characteristics of the (PMDC) motor. The model is simple and it has only two vehicle parameters K_a , K_v . The vehicle parameters used in simulation are predetermined considering floor slopes. The acceleration model is simulated in MATLAB for various floor slopes and simulation performance results are presented for the vehicle that is moving on floors like flat, upslope, down slope, etc. The vehicle parameters K_a and k_v are determined with consideration of various forces that are acting on the vehicle when it is moving on different slopes. These parameters can be used in mobile robot or automated guided vehicle (AGV) navigation and also for prediction of the vehicle path.

Key Words: Modelling, Torque-speed characteristics, Simulation.

1. INTRODUCTION

The electric vehicles including fuel-cell and hybrid vehicles have been developed very rapidly as a solution to energy and environmental problems. The selection of traction motors for the electric vehicle propulsion systems is a very important step that requires special attention. In fact, the automotive industry is still seeking for the most appropriate electric propulsion system. The propulsion system must have good efficiency, reliability and low cost. There are various types of electric motors; each motor has unique torque – speed characteristics. The tractive effort developed by the motor at given load can be determined from torque – speed characteristics of the motor. The performance of the electric vehicle mostly depends on the tractive effort developed by the motor at given inputs. The mathematical modeling of electric vehicle generally consists of two sub models i.e., tractive effort and vehicle acceleration. In this paper these two sub models are proposed for selected electric motor and electric vehicle with different floor slopes. The effort has been made to give simplified model for vehicle acceleration with only two vehicle parameters K_a and K_v . The parameters K_a and K_v are associated with torque-speed characteristics of selected PMDC motor and various forces that are acting on the vehicle at different floor slopes. The torque-speed characteristics graph of selected PMDC motor is first obtained with the help of set up shown in figure 1.3.

When the vehicle moves on a surface, various forces are acting on it. The forces may be in favour or opposite to the vehicle acceleration. The forces acting against the acceleration are rolling (friction between the tyres and surface), inertia and aerodynamic drag. The grade resistance force acts in favour of the acceleration when vehicle is moving on down slope and it acts opposite to the acceleration when vehicle is moving on up slope. In this paper small electric vehicle is used hence aerodynamic drag is not taken into account in vehicle acceleration modeling. Secondly, the vehicle speed is to be maintained low and it is assumed that the vehicle should operate on the floor of fixed surface conditions; hence, rolling resistance force is assumed to be constant.

2. RELATED WORK

R. Akcelikand et al., have presented a comparative study of different acceleration models on the data which has been obtained from real time traffic conditions. In their study, they found polynomial model was the best for prediction acceleration distance and fuel consumption [1]. Karen L. Butler et al., have been discussed a simulation and modeling package developed at Texas A&M University, V-Elph 2.01. V-Elph facilitates in-depth studies of electric vehicle (EV) and hybrid EV (HEV) configurations or energy management strategies. It also discusses the methodology for designing vehicle drive trains using the V-Elph package [2]. Muhammad IzharIshak el at., have analyzed the performance of four-wheel drive and independent steering on an over-steer in-wheel small eclectic vehicle by using numerical simulation. Two cornering conditions were simulated i.e. steady state cornering at below critical velocity and steady-state cornering over critical velocity [3]. H. Nasiri el at., developed a hybrid electric vehicle model based on United States Department of Energy Reports and used as a benchmark for comparison. Two different power dividers in different series-parallel topologies for hybrid electric vehicles have been compared [4]. M. L. Dou el at., have developed a control-oriented drivability model for an electric vehicle. They have used least square method to identify model parameters based on the data obtained from areal eclectic vehicle [5]. Different methods in vehicle mass and road grade estimation are discussed in [6], while real time road grade estimation using onboard sensor with GPS sensor is given in [7]. Muhammad Nasiruddin Mahyuddin el at., have presented observer-based parameter estimation scheme with sliding mode term has been developed to estimate the road gradient and the vehicle weight using only the vehicle's velocity and the driving torque [8]. Jose C. Pascoa el at., proposed an alternative approach to the standard on-road coast down test used in automotive industry and in ground vehicle research [9].

3. MODELLING

The electric motor is the key component of the electric vehicle. Generally, a Permanent Magnet Direct Current (PMDC) motor is used to drive all small electric vehicle. This type of motor is very widely used in applications such as portable tools, toys, electrically operated windows in cars, and small domestic appliances such as hair dryers, etc. M.S. Widyān el at., presented the dynamical model and analysis of DC shunt, series and permanent-magnet (PM) motors fed by photovoltaic (PV) energy systems at different illuminations [10]. The characterization of small DC brushed and brushless motors can be found in [11]. The classical DC electric motor is shown in Figure 1.1. The Permanent magnet DC brushed motors (PMDC motors) consist of permanent magnets, located in the stator, and windings, located in the rotor. The ends of the winding coils are connected to commutator segments that make slipping contact with the stationary brushes. Brushes are connected to DC voltage supply across motor terminals. Change of direction of rotation can be achieved by reversal of voltage polarity. The current flow through the coils creates magnetic poles in the rotor that interact with permanent magnet poles. In order to keep the torque generation in same direction, the current flow must be reversed when the rotor north pole passes the stator south pole. For this the slipping contacts are segmented. This segmented slip ring is called Commutator. The commutator segments are made from copper. The brushes are made from precious metal or carbon (graphite brush).

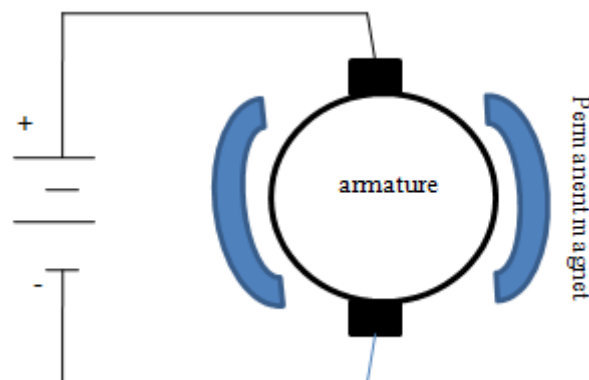


Figure 1.1: Basic construction of PMDC motor

3.1 Modelling Tractive Effort

The electric motor used has one coil, and the current passing through the wire near the magnet causes a force to be generated in the coil. If a wire in an electric motor has a length l meters, carries a current I amperes, and is in a magnetic field of strength B Wb.m^{-2} , then the force on the wire is:

$$F = BIL \quad (1)$$

If the radius of the coil is r , and the armature consists of n turns, then the motor torque T is given by the equation

$$T = 2nrBIL \tag{2}$$

The total flux passing through the coil, the term 2Blr can be replaced by ϕ , this gives:

$$T = n\phi I \tag{3}$$

Equation (3), gives the peak torque, when the coil is fully in the flux, which is perfectly radial. In practice, this will not always be so. The constant K_m connects the average torque with the current and the magnetic flux. The value of K_m depends on the number of turns in each coil, but also on the number of pole pairs, and other aspects of motor design

$$T = K_m\phi I \tag{4}$$

Equation (4), shows that the motor torque is directly proportional to the armature current (I). This armature current is depending on supply voltage (E_s) and armature resistance (R_a). The armature current in terms of supply voltage and back emf (E_b) is given by;

$$I = \frac{V}{R_a} = \frac{E_s - E_b}{R_a} = \frac{E_s}{R_a} - \frac{K_m\phi}{R_a} \omega \tag{5}$$

Put it in (4)

$$T = \frac{K_m\phi E_s}{R_a} - \frac{(K_m\phi)^2}{R_a} \omega \tag{6}$$

This important equation shows that the torque from this type of motor has a maximum value at zero speed, when stalled, and it then falls steadily with increasing speed. In this analysis, we have ignored the losses in the form of torque needed to overcome friction in bearings, and at the commutator, and windage losses. This torque is generally assumed to be constant, which means the general form of (6) still holds true, and gives the characteristic graph in figure 1. 2.

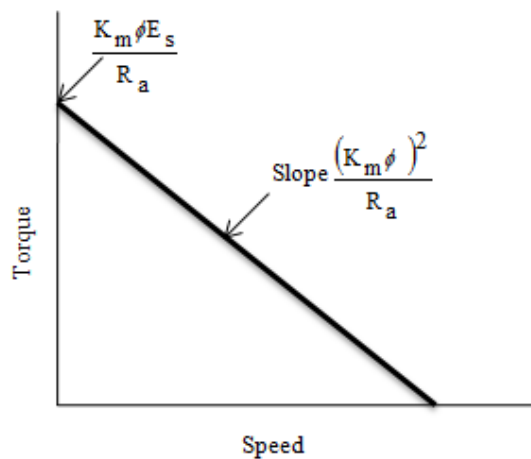


Figure 1.2: Torque –Speed graph for brushed PMDC motor

The set up shown in figure1.3 has been developed for determining the torque-speed characteristics of the motor that is used in the test vehicle. The (PMDC) motor is used in the test vehicle to apply tractive effort on the wheel to overcome vehicle resistance forces and to accelerate the vehicle. The procedure for development of equation for applied torque (tractive effort) on the vehicle wheel from torque-speed characteristics of PMDC motor is explained in section 3.1. The set up was fabricated and used to determine the torque-speed characteristics of the PMDC motor. The motor used in the test vehicle is small; hence this fabricated experimental set up is giving good results for all applied voltages and motor speeds and the setup is sufficient to determine maximum torque that motor can develop and maximum speed at no torque. For speed measurement contactless tachometer as shown in figure 1.3 (c) was used. In simulation model, the main inputs are; the maximum torque developed by the motor and maximum linear speed of the vehicle on application of that torque. Therefore, by knowing the maximum torque, the maximum angular speed of the motor, gear ratio and vehicle wheel radius, the linear speed of the vehicle on the ground or floor can be determined and used for simulation.

The relation between motor torque ($T_{motor}=T_{shaft}$), angular speed ω , maximum torque (T_{max}) and some constant K can be written as given by (7). This is basic equation of motor which gives torque developed by the motor at applied voltage with angular speed (ω). The constant K is the slope of torque – speed characteristics of the motor

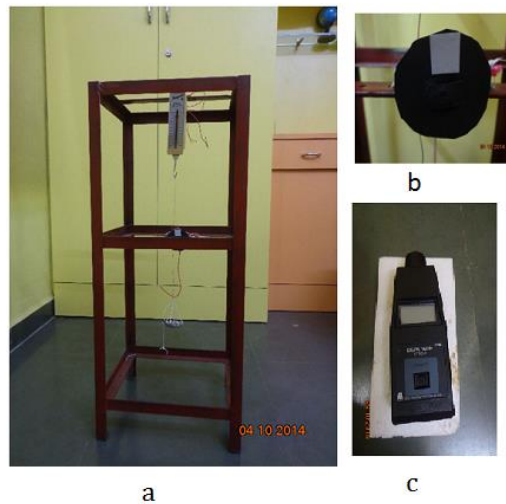


Figure 1.3: a] Motor Torque – speed characteristics set up consisting of testing structure, spring balance, load pan, motor fixing arrangement at middle of the frame; [b] Speed measurement arrangement, black circular marker with silver reflective tape enable to sense to the contactless digital tachometer and c] Contactless digital tachometer

This is basic important equation which gives the input for formulation of simulation model for electric vehicle

$$T_m = T_{max} - K\omega \tag{7}$$

Equation (7) can be modified as;

$$T_w = T_{max} - \frac{KG}{r} v \tag{8}$$

Equation (8) can be simplified as ;

$$T_w = T_{max} - K_m v \tag{9}$$

Now (8) gives the tractive effort (Nm) developed by the motor which is applied on the wheels of the electric vehicle to accelerate it. The tractive force can be obtained from (9) by dividing tractive effort to wheel radius (r). The relation between tractive force and vehicle linear speed (V) assuming ideal conditions is presented in figure 1.4. The E_{s1} and E_{s2} indicates the supply voltage.

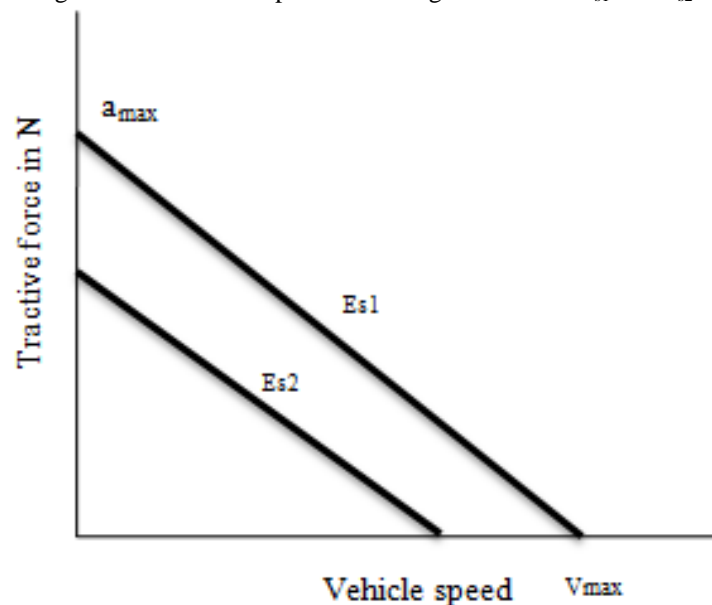


Figure 1.4: Tractive force Vs vehicle speed graph for applied voltage

3.2 Modelling Motor Vehicle Acceleration

A problem with vehicle kinematics models is that by empirically developing mathematical expressions that describe the acceleration patterns of the vehicle, the actual components that affect the motion of the vehicle; the tractive force provided by the prime mover (PMDC motor) and the resistance forces opposing the vehicle’s motion are not modeled explicitly. Therefore, these models are difficult to calibrate and do not generally provide a good fit to field data for each of the acceleration, speed, distance, and time domains. They also do not account for different vehicle types, roadway grades, and other factors that affect the vehicle acceleration patterns [12, 13]. To better account for an accelerating vehicle’s actual physics of motion, a number of acceleration models have been developed based on vehicle dynamics. These models characterize the acceleration of a vehicle based on the power output from the vehicle’s prime mover and power losses resulting from internal mechanical friction losses and external resistance forces, such as air resistance, rolling resistance and grade resistance forces.

In this paper, the torque- speed characteristics of the PMDC motor is used as a base for the development of acceleration model of electric vehicle. The primary equation proposed for vehicle acceleration is given as;

$$a_v = a_{\max} - \frac{a_{\max}}{v_{\max}} v \tag{10}$$

Where

- a_{\max} : Maximum acceleration produced by the vehicle
- v_{\max} : Maximum speed attained by the vehicle
- a_v : Vehicle acceleration at applied force F_t
- v : Speed of the vehicle at a_v

Put

$$K_a = a_{\max};$$

$$K_v = a_{\max}/v_{\max};$$

Equation (10) can be written as;

$$a_v = K_a - K_v V \tag{11}$$

Equation (11) can be written as;

$$\frac{dV}{dt} = K_a - K_v V \tag{12}$$

After simplification (12), we can write as;

$$\frac{dV}{K_a - K_v V} = dt \tag{13}$$

Integrating (13) and putting initial conditions, velocity of the vehicle at any time ‘t’ can be calculated by using (14).

$$V = \frac{K_a}{K_v} \left(1 - e^{-tK_v} \right) + V_0 e^{-tK_v} \tag{14}$$

Integrating (14) and putting initial conditions, distance covered by the vehicle at any given time, t can be calculated using (15).

$$S = \frac{K_a}{K_v} t - \frac{K_a}{K_v^2} \left(1 - e^{-tK_v} \right) + \frac{V_0}{K_v} \left(1 - e^{-tK_v} \right) \tag{15}$$

4. ACCELERATION MODEL SIMULATION

The acceleration model developed in section 3.2 is used for simulation. Equation (11) is integrated to obtain motor vehicle velocity and distance covered for given simulation time. The vehicle parameters K_a and K_v are determined by considering floor slopes and using data given in Table 1. The parameters calculated for given floor slope are only useful that particular slope. If the floor slope is changed the vehicle parameters must be changed and need to recalculate. The motor vehicle acceleration model is simulated on three different floor or road slopes i.e., flat slope, up slope and down slope. The motor vehicle parameters estimated

beforehand is given in table 1. The simulation results are presented in following sections 4.1, 4.2 and 4.3. The numerical solutions of (11), (14) and (15) are used for simulation in MATLAB.

Table 1 : Estimated vehicle parameters

Tractive force (F_t)	2.8 N
Rolling resistance force for flat floor	1.6 N
Grade resistance force for 3-degree floor slope	0.5 N
Vehicle mass (m)	1.02 Kg
Gear ratio(G)	9.21
Wheel radius (r)	0.024 m
Acceleration due to gravity (g)	9.81 m/s ²
Vehicle maximum velocity on flat floor (V_{max})	1.2 m/s
Vehicle maximum acceleration on flat floor (a_{max})	1.18
Vehicle parameter for flat floor (K_{af})	1.18
Vehicle parameter for flat floor (K_{vf})	0.98

4.1 Motor Vehicle on Flat Slope

When the vehicle is moving on flat floor or road the forces acting against the vehicle acceleration are rolling resistance force (friction between floor and tyre) and vehicle inertia. The rolling resistance force acting on the vehicle when it is on flat slope is already determined and given in Table 1. The vehicle parameters have been set considering all forces that are acting on the vehicle. For flat floor, the values of vehicle parameters used in simulation are $K_{af} = 1.18$, $K_{vf} = 0.98$. The vehicle acceleration model is simulated in MATLAB. The simulation time was 5 seconds and two vehicle range i.e., acceleration and cruising range are covered. The distance covered, velocity gained and acceleration of the vehicle are presented in figure 1.5. The vehicle reached at peak velocity after 4 seconds of acceleration where vehicle inertia force becomes zero.

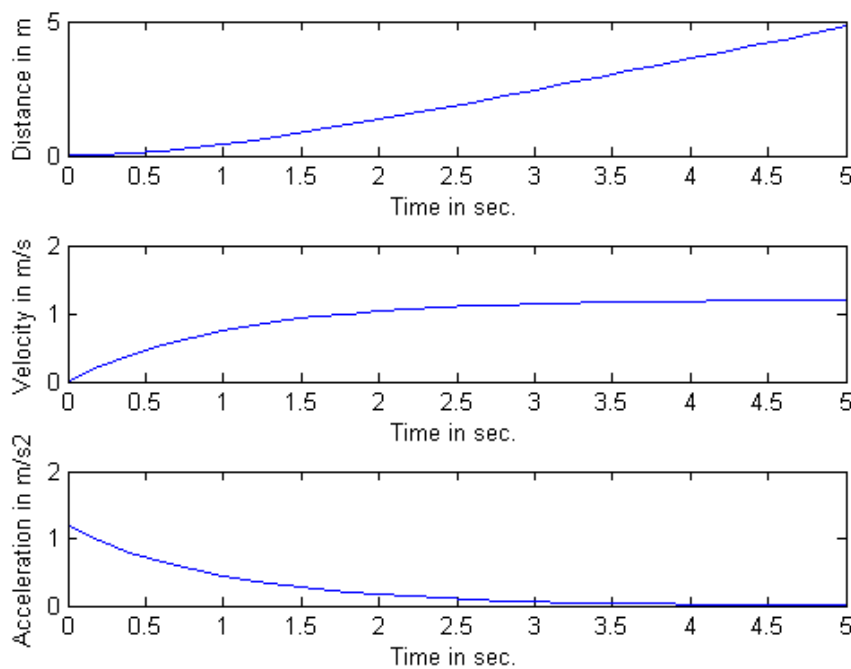


Figure 1.5: Display of simulation results of vehicle moving on flat slope

4.2. Motor Vehicle on up Slope

When the vehicle is moving on floor of up slope then the forces acting against the vehicle acceleration are rolling resistance, inertia and grade. Considering these forces, the vehicle parameters set for simulation are $K_{au}=0.68$, $K_{vu}=1.36$. These parameters are put in (11), (14), and (15) and results obtained are presented in figure 1.6, for vehicle distance covered, velocity and

acceleration. In this case, the vehicle reached at peak velocity after 3 seconds of acceleration where vehicle inertia force becomes zero.

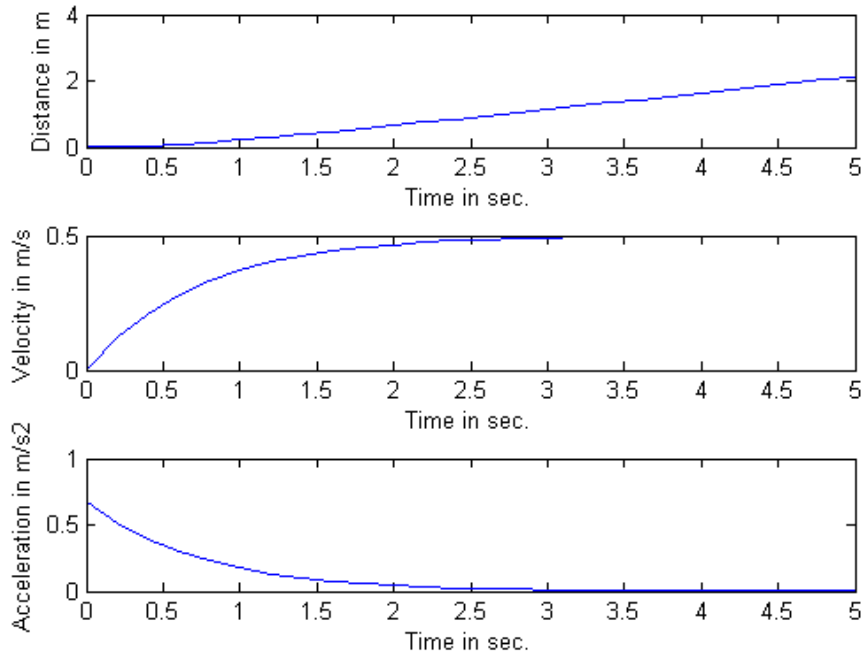


Figure 1.6: Display of simulation results of vehicle moving on upslope

4.3. Motor Vehicle on down Slope

When the vehicle is moving on floor of down slope then the forces acting against the vehicle acceleration are rolling resistance, inertia and in favour of acceleration is grade. Considering these forces, the vehicle parameters set for simulation are $K_{ad}=1.67$, $K_{vd}=1.11$. These parameters are put in (11), (14), and (15) and results obtained are presented in figure 1.7, for vehicle distance covered, velocity and acceleration. The vehicle reached at peak velocity after 5 seconds of acceleration where vehicle inertia force becomes zero.

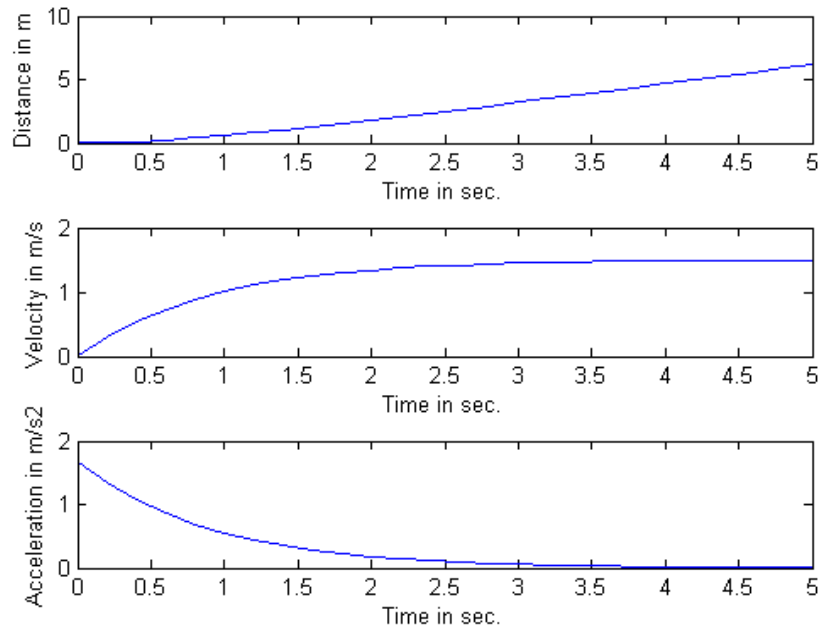


Figure 1.7: Display of simulation results of vehicle moving on downslope

4.4. Motor Vehicle Inertia Forces

The vehicle inertia force is obtained for three different floor slopes i.e., flat, up slope and down slope using vehicle parameters K_{af} , K_{vf} , K_{au} , K_{vu} , K_{ad} and K_{vd} , respectively. The inertia force with time for all these floor slopes are calculated and results are presented in figure 1.8. It is clear from the figure 1.8 that the inertia force is maximum at the start of the vehicle and it becomes zero when vehicle reaches its maximum speed. It is true for all floor slope conditions.

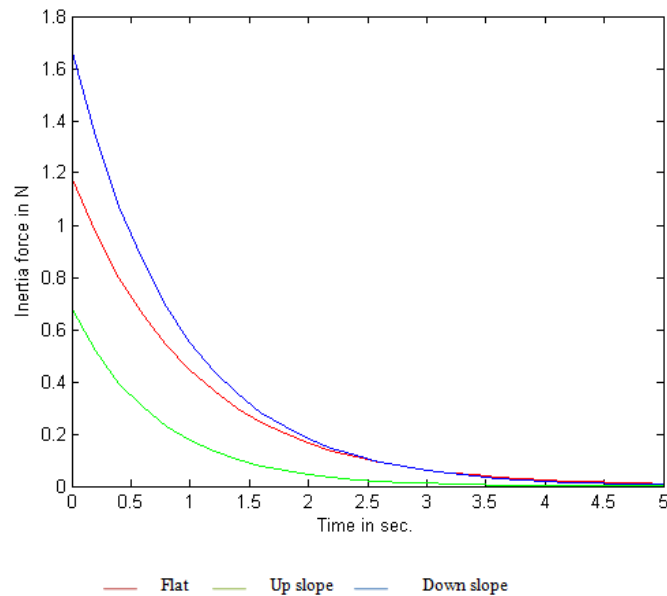


Figure 1.8: Display of inertia forces for different floor slopes

4.4. Results Comparison

The motor vehicle acceleration model presented in this paper is simulated in MATLAB for simulation time of 5 seconds each floor slope. The simulation covers two vehicle ranges i.e., acceleration and cruising. The distance covered by the vehicle during these ranges are shown in table 2. From table, it is clear that when the vehicle is on up slope it covers less distance during acceleration as compared to when it is on down slope. The total distance covered is less in it is moving on up slope for period of 5 seconds.

Table 2: Distance covered by the vehicle on different floor slopes

Floor slope	Distance during acceleration in m	Distance during cruising in m	Total distance
Flat slope	3.6	1.2	4.8
Up slope	1.14	0.99	2.13
Down slope	5.3	0.87	6.17

5. CONCLUSION AND DISCUSSIONS

This paper proposes the mathematical model for tractive effort from torque-speed characteristics of PMDC electric motor and vehicle acceleration. Initially, mathematical equation for vehicle acceleration was obtained and then it solved analytically for vehicle velocity and distance with respect to the time. In vehicle acceleration equation, only two parameters K_a and K_v are used. The test vehicle used for finding various forces and parameters K_a and K_v is a small electric car. The values of these parameters for floors like flat, up slope and down slope are given in section 4.1, 4.2 and 4.3. The simulation results are validated for the vehicle when it is moving on flat floor using overhead camera sensor. However, the low field of view of camera limits the working space; the results are not getting validated for up and down slope floors.

The floor slope changes the values of parameters of K_a and K_v , because of changes in forces that are acting on the vehicle. In this paper, the values of different forces of given test vehicle and floor slope are determined beforehand and then these values are used to set the values K_a and K_v . The simulation results obtained for different floor slopes are the indication of good proposed graphs for distance, velocity and acceleration for given set conditions. There is scope for further study of these models to find the values of the K_a and K_v using vehicle position data which further can be used for vehicle path prediction and control.

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