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The Power Factor Improvement Modelling with the Runge-Kutta 4th Order, Analytical, and Experimental Method

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ABSTRACT

The Power factor improvement modeling is conducted by modeling electrical circuits that have a low power factor, then the power factor will be improved by pairing capacitors in series and in parallel. The capacitance values of the connected capacitors are varied for maximum power factor value. The power factor improvement modeling was analyzed using the Runge-Kutta 4th Order (RK-4) method and the analytical calculations were then compared with the experimental method. The study shows the relationship between the value of the power factor to the capacitance value and the relationship between active power, apparent power, and reactive power to the capacitance value of capacitors installed in series or parallel in the circuit. The results of calculations using the RK-4 method, analytical methods and experimental data show the same pattern, with a relative error below 8%. The results of the analysis show that the parallel installation of capacitors is the most appropriate power factor improvement model.

Key Words: Analytical Method, Experimental Method, Power Factor, Runge-Kutta, Numerical Method.

1. INTRODUCTION

The power triangle in Figure 1 illustrates the relationship between active power (P), apparent power (S) and reactive power (Q). Between S and P are separated by an angle φ which is the same angle as the phase difference angle between voltage and current. The power factor is the ratio of P and S or the value of $\cos\varphi$. Improving the power factor is by making the angle φ closer to 0 or $\cos\varphi$ closer to 1, the improvement in the power factor occurs if S gets closer to P, it means that the magnitude of P will approach the magnitude of S. In cases where $\varphi = 0$, $\cos\varphi = 1$, $S = P$ means that all the apparent power provided by the source can be utilized as active power.

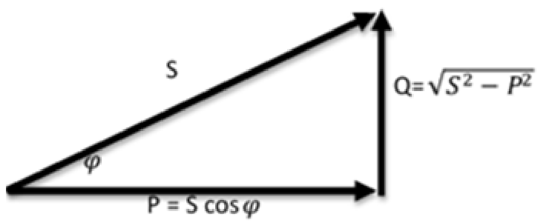

Figure 1. The relationship between active power (P), apparent power (S) and reactive power (Q)

One way to increase the value of the power factor of electrical installations and electrical networks in industry is by installing capacitor banks [1, 2]. Installing capacitors in series and parallel in the circuit can improve the power factor [3-5]. Capacitor installation can reduce reactive power (Q) so that the φ angle value is close to zero or the $\cos\varphi$ value is close to 1. The first stage of power factor improvement modeling is to model a circuit with a small inductive power factor value, namely the RL series circuit, then the capacitors are installed in series or parallel with varying capacitance values C. The circuit model is then analyzed numerically and analytically, then tested through experimental data collection.

2. LITERATURE SURVEY

2.1 Circuit Modeling

Improving the power factor using the electrical circuit modelling is shown in Figure 2. The resistive load in an electrical installation is represented by a resistor with a resistance of $R = 10\ \Omega$ installed in series with an inductive load represented by an inductor with an inductance $L = 54$ mH. In theoretical calculations the value of the power factor in the circuit $\cos\varphi = 0.507$. Improving the power factor using a capacitor with capacitance C , this is installed in series and installed in parallel. The value of C was varied to get the greatest power factor value.

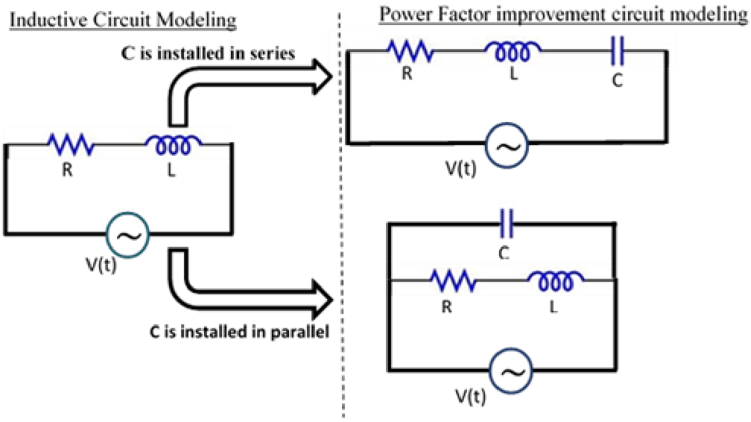

Figure 2. Power factor correction circuit modeling: (a) inductive circuit, (b) power factor correction circuit with C series installation, (c) power factor correction circuit with parallel C installation.

2.2 Runge-Kutta 4th Order Calculation

The differential equation for the RLC circuit in Figure 2 (b) is as follows:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V(t) \tag{1}$$

and the differential equation for the RLC circuit in Figure 2 (c) is as follows:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = \frac{q}{c} = V(t) \tag{2}$$

taking the form $V(t) = V_m \sin(\omega t)$.

Equation 1 and Equation 2 is an ordinary differential equation 2nd order, and both will be solved by the Runge-Kutta 4th Order method [6], [7], [8], [9], [10]. Then substitute $I = \frac{dq}{dt}$ and $Q = q$ by reducing differential equation 2nd order to 1st order into two equations as follows:

$$\frac{dI}{dt} = f(t, I, Q) \tag{3}$$

$$\frac{dQ}{dt} = u(t, I, Q) = I \tag{4}$$

So, by using the method of Runge-Kutta 4th order a set of equations would be:

$$I_{i+1} = I_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{5}$$

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$$Q_{i+1} = Q_i + \frac{h}{6}(l_1 + 2l_2 + 2l_3 + l_4) \quad (6)$$

In which

$$k_1 = f(t_i, I_i, Q_i) \quad (7)$$
$$k_2 = f\left(t_i + \frac{h}{2}, I_i + \frac{1}{2}k_1 h, Q_i + \frac{1}{2}l_1 h\right) \quad (8)$$
$$k_3 = f\left(t_i + \frac{h}{2}, I_i + \frac{1}{2}k_2 h, Q_i + \frac{1}{2}l_2 h\right) \quad (9)$$
$$k_4 = f(t_i + h, I_i + k_3 h, Q_i + l_3 h) \quad (10)$$
$$l_1 = u(t_i, I_i, Q_i) \quad (11)$$
$$l_2 = u\left(t_i + \frac{h}{2}, I_i + \frac{1}{2}k_1 h, Q_i + \frac{1}{2}l_1 h\right) \quad (12)$$
$$l_3 = u\left(t_i + \frac{h}{2}, I_i + \frac{1}{2}k_2 h, Q_i + \frac{1}{2}l_2 h\right) \quad (13)$$
$$l_4 = u(t_i + h, I_i + k_3 h, Q_i + l_3 h) \quad (14)$$

To get the power factor, the effective current (I_eff), active power (P), apparent power (S), reactive power (Q) should be calculated.

$$I_{eff} = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N}[I(i)]^2} \quad (15)$$
$$I_{R\,eff} = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N}[I_R(i)]^2} \quad (16)$$
$$P = I_{R\,eff}^2 \times R \quad (17)$$
$$S = V_{eff} \times I_{eff} \quad (18)$$
$$Q = \sqrt{S^2 - P^2} \quad (19)$$

So, the power factor will be $\cos\varphi = \frac{P}{S}$

The RK-4 method uses parameters $I(0) = 0$, $Q(0) = 0$, $h = 0{,}0002$ s, $V_{eff} = 20$ V, $L = 54$ mH, $R = 10\ \Omega$, $\omega = 2\pi f$, $f = 50$ Hz, $C = 10 - 1000\ \mu$F, the amount of literacy $N = 100$.

2.3 Analytical Calculations

The impedance Z for the series RLC circuit in figure 2 (b) is derived from:

$$Z = R + (XL - XC)i \quad (20)$$

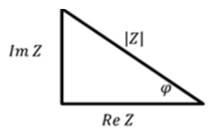

Figure 3. *Z* complex number triangle

Where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$, the impedance is obtained,

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \quad (21)$$

Then we get the power factor ($\cos\varphi$) from the comparison of the value of Re Z with $|Z|$.

$$\cos\varphi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad (22)$$

and I_{eff} is obtained from,

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$$I_{eff} = \frac{V_{eff}}{|Z|} = \frac{V_{eff}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{23}$$

Active power (P), apparent power (S) and reactive power (Q) are obtained from the equation:

$$P = I_{eff}^2 R = {V_{eff}}^2 \frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{24}$$

$$S = V_{eff} \times I_{eff} = \frac{{V_{eff}}^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{25}$$

$$Q = \sqrt{S^2 - P^2} = {V_{eff}}^2 \sqrt{\left(\frac{1}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} - \frac{R^2}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^2}\right)} \tag{26}$$

Whereas the RLC circuit in Figure 2 (c) has an impedance equation:

$$\frac{1}{Z} = \frac{1}{R + i\,X_L} - \frac{1}{iX_C} \tag{27}$$

The equation is simplified into the form $Z = Re\,Z + i\,Im\,Z$ to become:

$$Z = \frac{{X_C}^2}{R^2 + (X_L - X_C)^2}\left(R + i(X_L - \frac{R^2 + {X_L}^2}{X_C})\right) \tag{28}$$

Then the impedance is obtained:

$$|Z| = \frac{{X_C}^2}{R^2 + (X_L - X_C)^2}\sqrt{R^2 + \left(X_L - \frac{R^2 + {X_L}^2}{X_C}\right)^2} \tag{29}$$

Then we get the power factor ($\cos\varphi$) from the comparison of the value of $Re\,Z$ with $|Z|$,

$$cos\,\varphi = \frac{R}{\sqrt{R^2 + (\omega L - (R^2 + \omega^2 L^2)\omega C)^2}} \tag{30}$$

and I_{eff} is obtained from,

$$I_{eff} = \frac{V_{eff}}{|Z|} = V_{eff}\left(\frac{(\omega CR)^2 + \left(1 - \omega^2 LC\right)^2}{\sqrt{R^2 + (\omega L - (R^2 + \omega^2 L^2)\omega C)^2}}\right) \tag{31}$$

Active power (P), apparent power (S) and reactive power (Q) are obtained from the equation:

$$P = I_{R\,eff}^2 R = {V_{eff}}^2 \frac{R}{R^2 + (\omega L)^2} \tag{32}$$

$$S = V_{eff} \times I_{eff} = {V_{eff}}^2 \left(\frac{(\omega CR)^2 + \left(1 - \omega^2 LC\right)^2}{\sqrt{R^2 + (\omega L - (R^2 + \omega^2 L^2)\omega C)^2}}\right) \tag{33}$$

$$Q = \sqrt{S^2 - P^2} = {V_{eff}}^2 \sqrt{\left(\left(\frac{(\omega CR)^2 + (1 - \omega^2 LC)^2}{\sqrt{R^2 + (\omega L - (R^2 + \omega^2 L^2)\omega C)^2}}\right)^2 - \left(\frac{R}{R^2 + (\omega L)^2}\right)^2\right)} \tag{34}$$

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Analytical calculations are derived from $V_{eff} = 20$ V, $L = 54$ mH, $R = 10\ \Omega$, $\omega = 2\pi f$, $f = 50$ Hz (Government Electric Company frequency) and $C = 10 - 1000$ µF.

3. OBJECTIVE OF RESEARCH

The objective of this research is to investigate the influence of capacitor capacitance values on power factor improvement and to determine the appropriate electrical circuit model for experimental activities on power factor improvement at the laboratory scale.

4. RESEARCH METHODOLOGY

This research uses numerical (Runge-Kutta 4th Order), analytical and experimental methods. The first step is created simple inductive circuit model (RL), then the capacitors are installed in series or parallel to improve the power factor. The differential equation of the circuit model is solved using numerical method (Runge-Kutta 4th order) and analytical method. The calculation results of both numerical and analytical methods are compared with experimental data. The research flowchart can be seen in Figure 4.

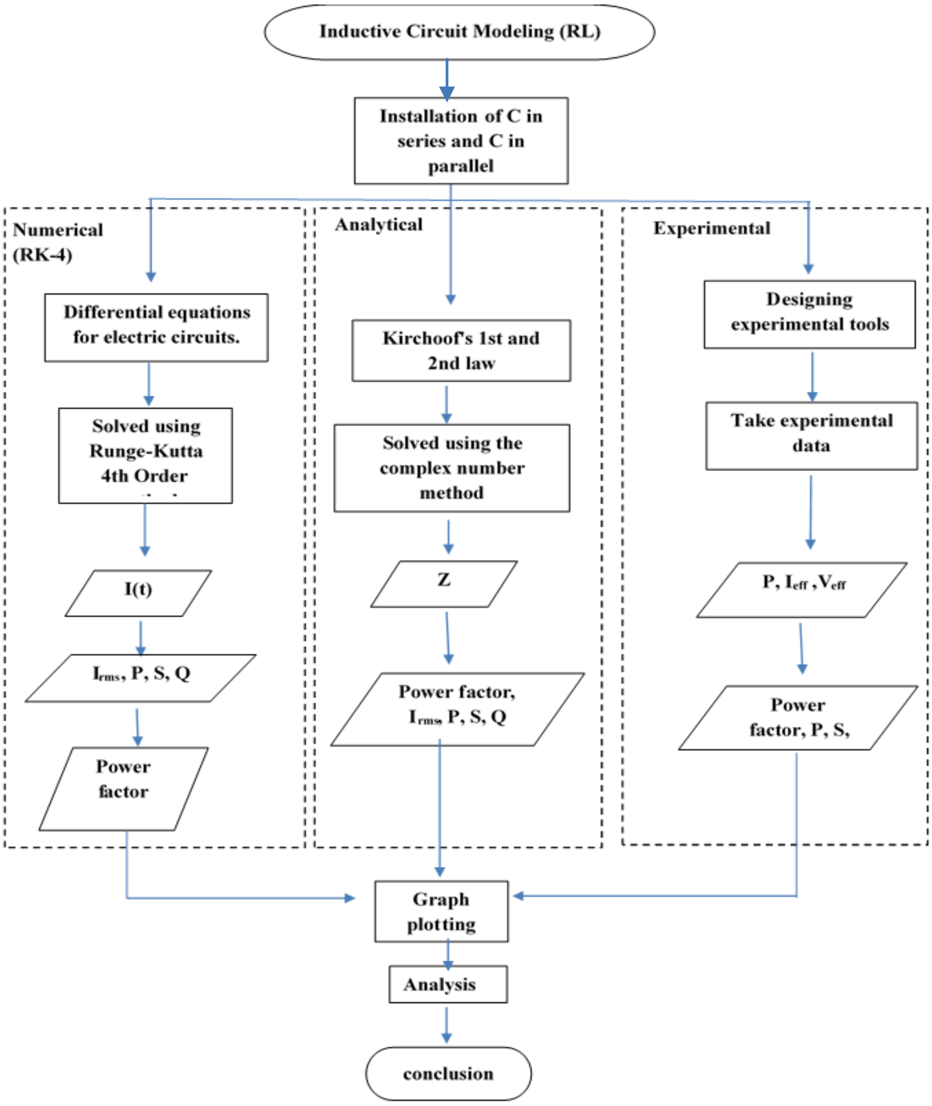

Figure 4. Research flowchart

In Figure 5, the experimental set to measure the power factor value consists of resistors, inductors, capacitors, ammeters, voltmeters, wattmeters, and AC power supplies. The 10 Ω resistors, 54 mH Inductors and Capacitors are connected as shown in Figure 1(b) and 1(c) with capacitor C values varied at 25 µF, 50 µF, 100 µF, 200 µF, 250 µF,

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300 μF, 333.33 μF, 400 μF, 500 μF and 1000 μF. The wattmeter is used to measure Active Power (P), while the Ammeter and Voltmeter are used to measure Artificial Power ($S = V_{eff} \times I_{eff}$), while reactive power is calculated from the equation $Q = \sqrt{S^2 - P^2}$. The power factor of the experiment results is the ratio of Active Power (P) to False Power (S).

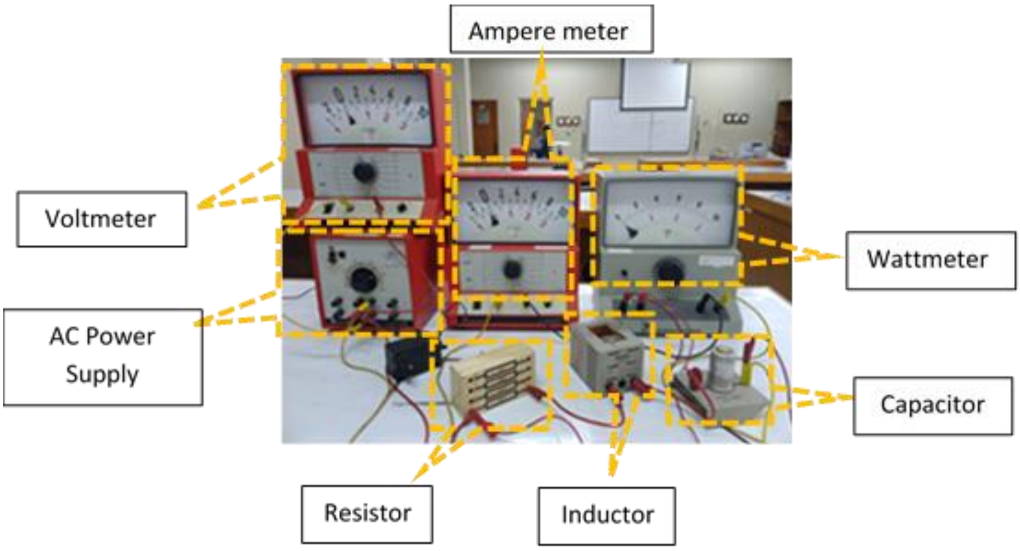

Figure 5. Set of RLC circuit Power Factor measurement experiment tools

5. RESULT AND DISCUSSION

The inductive circuit (Figure 2 (a)) based on the calculation of the analytical method and the numerical method (RK-4) has a value of $\cos\varphi = 0{,}5078$, while based on experimental data it has a value of $\cos\varphi = 0{,}56$. By calculating RK-4 using MATLAB R210a software, the current and voltage forms are obtained in Figure 6.

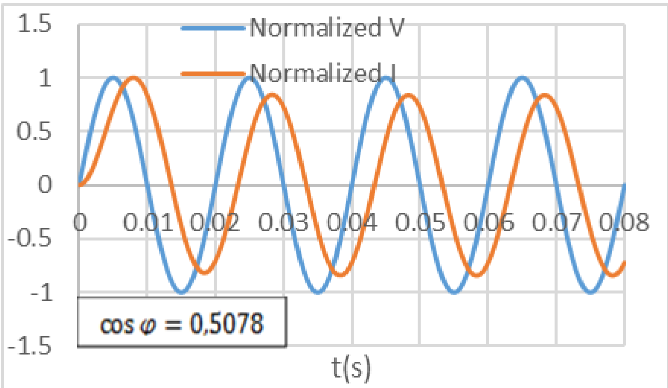

Figure 6. The phase difference between the current and the voltage in the circuit is inductive in Figure 1 (a) with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on the RK-4 method.

In Figure 6 shows that the current is lagging against the voltage having $\cos\varphi = 0{,}5078$, this power factor will be improved by the addition of capacitors in series and parallel.

Figure 7 shows that with the addition of the capacitor C value in series, the phase difference between the current and the voltage changes. Initially with the increase in C, the current and voltage phase difference decreases and the current becomes leading to the voltage, until at $C = 187.63$ μF, the current and voltage have the same phase, meaning $\cos\varphi = 1$. The value of $C = 187.63$ μF is the value of C needed for resonance to occur ($XC = XL$ or $C = \frac{1}{\omega^2 L}$). Nevertheless, after passing the resonant condition the current becomes lagging with respect to the voltage and the phase difference between the voltages increases again with the increase in the value of C.

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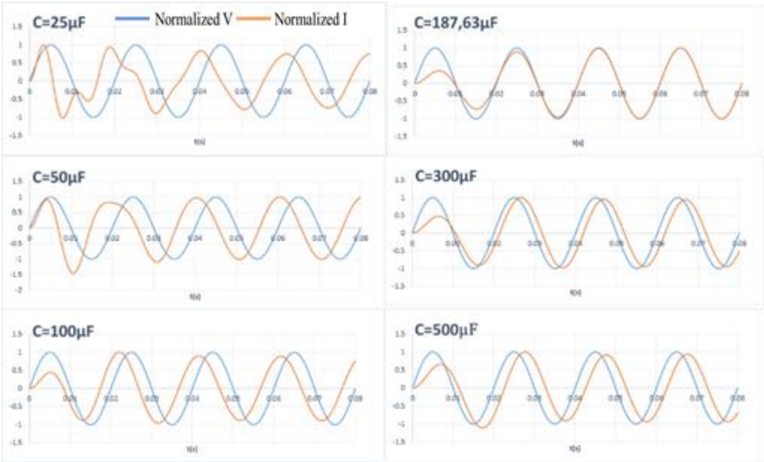

Figure 7. Changes in the phase difference between current and voltage by varying the value of C in series with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on the RK-4 method.

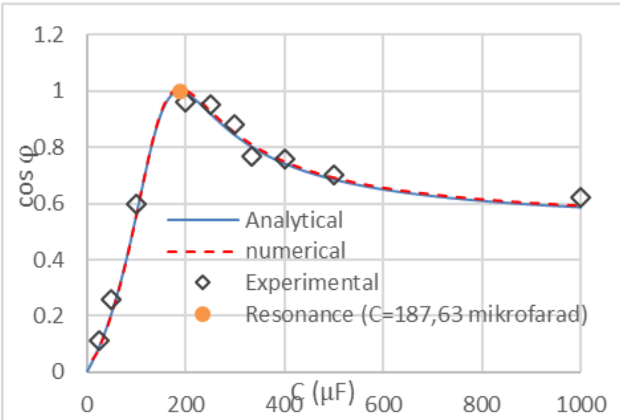

Figure 8. Graph of Power Factor (cos φ) against the value of C in the capacitor installation in series with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on the numerical method (RK-4), analytical and experimental.

Figure 8 shows the relationship between the power factor value and the capacitor C value in series. Based on the results of numerical calculations (RK-4), Analytical Calculations and Experimental Results show the same pattern. Initially, with the increase in C, the power factor value will get bigger, numerically, and analytically at $C = 187.63$ μF the power factor is the maximum value. However, after passing through the resonance conditions the power factor will decrease with increasing value of C.

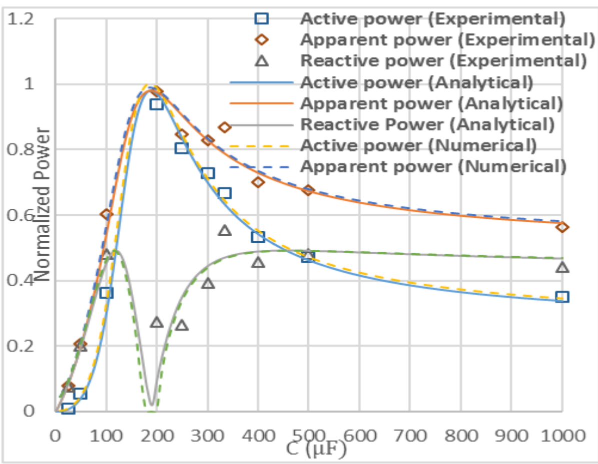

Figure 9. Graph of active power, apparent power, and reactive Power to the value of C in the capacitor installation in series with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on numerical methods (RK-4), analytical and experimental.

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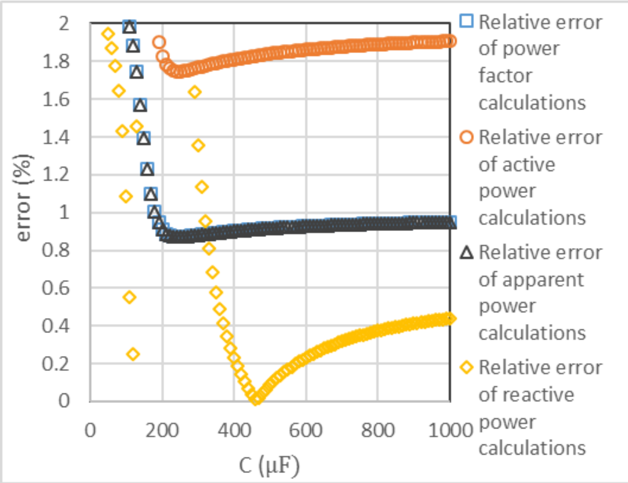

Figure 10. The relative error of calculating the numerical method with the analytical method to the value of C in the capacitor installation in series.

Figure 9 shows the relationship between active power, apparent power, and reactive power to the capacitor C value in series. Based on the results of numerical calculations (RK-4), analytical calculations and experimental results show the same pattern. Initially, with the increase in C, active Power and apparent power will get bigger until active power and apparent power will have the same value at their maximum value, namely at $C = 187.63$ μF which causes an improvement in the power factor value $\cos\varphi = 1$. After that active power and apparent power will decrease again at a different speed to C, so that active Power is always smaller than the apparent power and the Power Factor will decrease. Based on Figure 9, the decrease in reactive power in capacitor installation in series is caused by an increase in active power accompanied by an increase in apparent power, although the power factor increases but this results in an increase in active power and current in the load so that it is ineffective. Installing capacitors in series is not a solution to improving the power factor of an inductive circuit.

The relative error in the calculation of the numerical method against the analytical calculation has an average value of 1.98%. The relative error for the change in the value of C is given in Figure 10, while the average relative error of the numerical method calculation with experimental results is 7.10% and the relative error between the experimental results and the analytical calculation is 6.81%.

Figure 11 shows that the addition of the capacitor C value in parallel makes the phase difference between the current and the voltage change. Initially with the increase in C, the phase difference between the current and the voltage decreases and the current lagging against the voltage, until at $C = 139.25$ μF, the current and voltage have the same phase, meaning $\cos\varphi = 1$. The value of $C = 139.25$ μF is the value of C needed for resonance to occur ($C = L/(R^2 + \omega^2 L^2)$). However, after passing the resonant condition the current becomes leading to the voltage and the phase difference between the voltages. increases again with increasing value of C.

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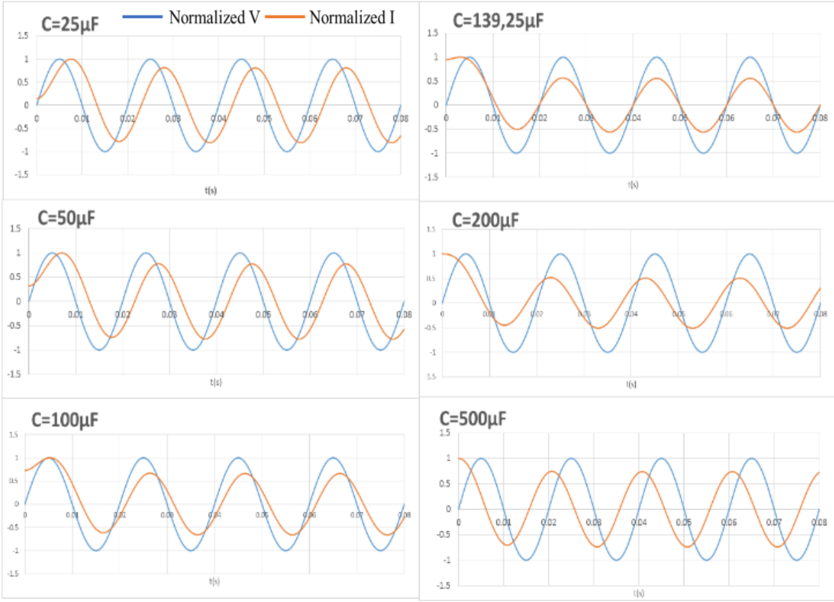

Figure 11. Changes in the phase difference between current and voltage by varying the value of C in parallel with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on the RK-4 method.

Figure 12 shows the relationship between the power factor value and the capacitor C value in parallel. Based on the results of numerical calculations (RK-4), analytical calculations and experimental results show the same pattern. Initially, with the increase in C, the power factor value will get bigger, numerically, and analytically at $C = 139.25$ μF the power factor is the maximum value. However, after passing through the resonance conditions the power factor will decrease with increasing value of C.

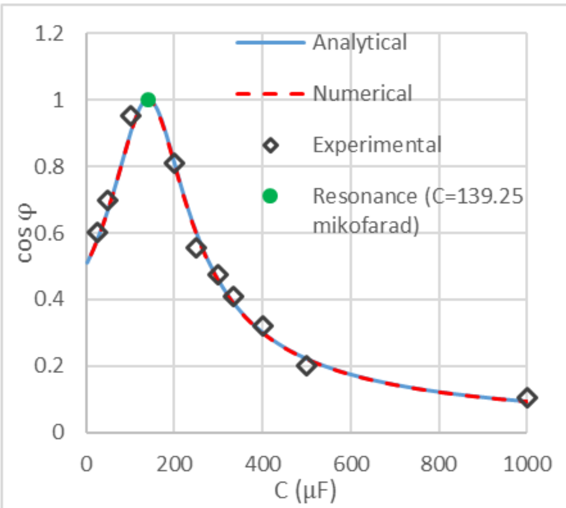

Figure 12. Graph of Power Factor ($\cos \varphi$) against the value of C on the capacitor installation in parallel with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on numerical methods (RK-4), analytical and experimental.

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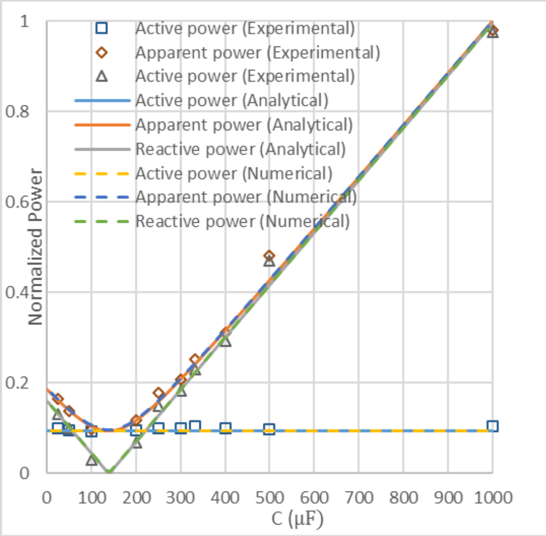

Figure 13. Graph of active power, apparent power, and reactive Power to the value of C in the capacitor installation in parallel with $R = 10\ \Omega$, $L = 54$ mH, and $f = 50$ Hz based on numerical methods (RK-4), analytical and experimental.

Figure 13 shows the relationship between active power, apparent power, and reactive power to the value of capacitor C in parallel. Based on the results of numerical calculations (RK-4), analytical calculations and experimental results show the same pattern. Based on numerical and analytical calculations, the active power does not change with the increase in the value of C. The apparent power decreases with the increase in the value of C until the value is the same as the active Power at resonance, namely at $C = 139.25\ \mu\text{F}$ which causes an improvement in the power factor $\cos\varphi = 1$, after that active power will increase with respect to C. The decrease in reactive power in the parallel installation of capacitors is caused by a decrease in apparent power while the active power is relatively unchanged, so that the load still gets optimal power and current when the power factor is improved. Electrical power will be used efficiently due to reduced apparent power and reduced reactive power. Installing capacitors in parallel is a solution to improve the power factor of an inductive circuit.

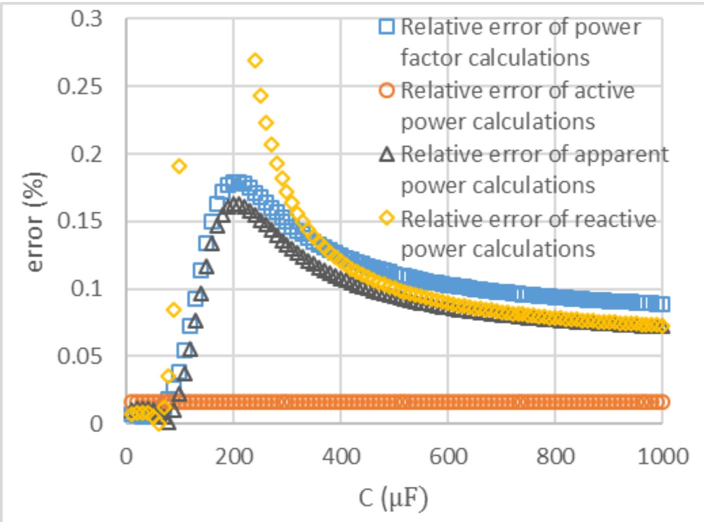

Figure 14. Relative error of numerical method calculation with analytical method to the value of C in parallel capacitor installation.

The relative error of the numerical calculation method against the analytical calculation has an average value of 0.07%. The relative error of the change in the value of C is given in Figure 15, while the average relative error of the numerical method calculation with experimental results is 5.26% and the relative error between the experimental results and the analytical calculation is 6.31%.

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6. CONCLUSION

Based on the results of Runge-Kutta 4th Order calculations, analytical calculations and experimental results show that adding capacitors in series or parallel to an inductive circuit (low power factor), can improve the value of the power factor. Improving the power factor value by installing capacitors in parallel is more rational than installing capacitors in series. Improvement of power factor by installing capacitors in parallel, namely by reducing the apparent power value to close to the active power value, even though the circuit is given additional capacitors, the active power value is always constant, so that the power absorbed by the electrical components is always optimal. Meanwhile, improving the power factor by installing capacitors in series is by increasing the apparent power value until it approaches the active power value when it reaches the maximum value, and adding capacitors to the circuit makes the active power value enlarge, so that the power absorbed by the electrical components will overload (can damage electrical components).

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