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A Forty-Two Points Second Order Rotatable Design in Three Dimensions Constructed using Trigonometric Functions Transformations

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ABSTRACT

In this study, a new forty-two point's second order rotatable design is constructed using transformations of some trigonometric functions. This design permits a response surface to be fitted easily and provides spherical information contours besides the economic use of scarce resources in relevant production processes.

Key words: Response Surface, Rotatable Designs, Second Order.

1.0 INTRODUCTION

In any experimental work, it is important to choose the best design in a class of existing designs. The choice is solely dependent of the interest of the experimenter and the adequacy of an experimental design that can be determined from the information matrix, Box [10]. The technique of fitting a response surface is one widely used to aid in the statistical analysis of experimental work in which the yield of a product depends in some unknown fashion on one or more controllable variables, Bose and Draper [1]. Before the details of such an analysis can be carried out, experiments must be performed at predetermined levels of controllable factors, i.e. an experimental design must be performed prior to experimentation. Box and Hunter [9] suggested designs of a certain type which they called rotatable as being suitable for such experimentation. They developed second order rotatable designs through geometrical configurations. Box and Wilson [11] pointed out that the technique of fitting a response surface is one widely used to aid in the statistical analysis of experimental work in which the response of the product depends in some unknown fashion, on one or more controllable variables. Mutiso [5] constructed specific optimal second order rotatable designs in three, four and five dimensions. Koske *eta*l. [6,7,8] constructed optimal second order rotatable designs and gave practical hypothetical examples. Cornelious [2,3,4] constructed optimal sequential third order rotatable designs in three four and five dimensions, and a second order rotatable design of 33 and 39 points respectively in three dimensions giving practical hypothetical examples. The current study gives a new 42-Points second order rotatable design in three dimensions constructed using Trigonometric functions.

2.0 METHODS

2.1 Moment conditions for second order rotatability

Suppose a second order response surface is to be fitted, then the following model is ideal;

$$y_u = \beta_0 + \sum_{u=1}^{N} \beta_i x_{iu} + \sum_{u=1}^{N} \beta_{ii} x_{iu}^2 + \sum_{u=1}^{N} \beta_{ij} x_{iu} x_{ju} + e_u \qquad (1)$$

Where x_i denotes the level of the i^{th}factor (i=1,2,3…k) in the u^{th} run, (u=1,2,3…, N) of the experiment and e_u is the un correlated random error with mean zero and variance σ^2.

According to Box and Hunter [9], A second order response surface is achieved if the design points satisfy the following moment conditions.

i. $\sum_{u=1}^{N} x_{iu}^2 = N\lambda_2$ (2)

ii. $\sum_{u=1}^{N} x_{iu}^4 = 3N\lambda_4$ (3)

iii. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = N\lambda_4$ for ($i \neq j = 1,2,\dots,k$) and $3, \lambda_2, \lambda_4$, are constants.

iv. $\sum_{u=1}^{N} x_{iu}^4 - 3\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 0$ (4)

v. All the other odd order moments are zero.

If all the above conditions are satisfied, then the set of points is said to form a rotatable arrangement.

2.2 Non-Singularity Conditions

Consider the following condition as given by Box and Hunter [9],

$$\frac{\lambda_4}{\lambda_2^2} > \frac{k}{k+2} \quad (5)$$

Where k is the number of factors which in this case will be (3, 4,…,k) and, λ_2, and λ_4 are obtained from the moment conditions given in (2) and (3) above.

If 4 and 5 are satisfied, then the set of points is said to make a rotatable design.

2.3 Rotatable Arrangements from Trigonometric Functions transformations (6s points)

Consider the following transformations,

$$T_1 = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & -1 \end{bmatrix}, \; T_2 = \begin{bmatrix} \cos\frac{\lambda}{2} & -\sin\frac{\lambda}{2} & 0 \\ \sin\frac{\lambda}{2} & \cos\frac{\lambda}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (6)$$

Where $\lambda = \frac{2\pi}{s}$ and ≥ 4 , Bose and Draper [1].

The transformations in (6) are applied to the points of the form (r,0,b) i.e points on the plane $y = 0$ and all other points obtained from repeated replications on T_1 and T_2

The permutation group (I, W, W^2) generated by ;

$W = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is also applied to (r,0,b) to give

$T_z(r,0,b)$, $T_x(r,0,b)$, and $T_y(r,0,b)$, (7)

These gives 6s points which we denote by T(r,0,b) and with co-ordinates.

S$(rcos\, t\lambda, rsin\, t\lambda, b)$

S$(rcos\left(t+\frac{1}{2}\right)\lambda, rsin\left(t+\frac{1}{2}\right)\lambda, -b)$

S$(b, rcos\, t\lambda, r\sin t\lambda)$

S$(-b, rcos\,(t+\frac{1}{2})\lambda$, $\text{rsin}(t+\frac{1}{2})\lambda$ (8)

S$(rcost\lambda, b, rsint\lambda)$

S$(rcos\left(t+\frac{1}{2}\right)\lambda, -b,\ rsin\left(t+\frac{1}{2}\right)\lambda)$

Where $t = 0,1,2,\dots(s-1)$

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Applying the conditions in (2,3,4,and 5), the set T(r,0,b) gives sums of powers and products as;

$$\sum_{u=1}^{N} x_{iu}^2 = 2s(r^2 + b^2) \quad (9)$$

$$\sum_{u=1}^{N} x_{iu}^4 = \frac{sr^2(3r^4+4b^4)}{2} \quad (10)$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \frac{sr^2(r^2+8b^2)}{4} \quad (11)$$

And all the other powers and products up to and including order four are zero. Using (4), it follows that,

$$\sum_{u=1}^{N} x_{iu}^4 - 3\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \frac{s(3r^4-24r^2b^2+8b^4}{4} \quad (12)$$

The excess of each single point varies in each case and it is necessary to consider the total effect over all points since its excess can be made positive or negative according to the choice of r and b it will be possible to combine the set T (r,0, b) with sets of both positive and negative excesses.

2.4 Rotatable arrangement for a Set of 12 Points

Consider another set of 12 points denoted by $s(p, p, 0)$

The set of points is summarized by the table 1 runs, in the table below.

Table.1 The set of points

Run	x_1	x_2	x_3
1	P	P	0
2	-P	P	0
3	P	-P	0
4	-P	-P	0
5	P	0	P
6	-P	0	P
7	P	0	-P
8	-P	0	-P
9	0	P	P
10	0	-P	P
11	0	P	-P
12	0	-P	-P

(13)

Applying 2,3, and 4, on the above sets of points gives;

$\sum_{u=1}^{N} x_{iu}^2 = 8p^2$,$\sum_{u=1}^{N} x_{iu}^4 = 8\,p^4$,$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 4p^4$ and

$$\sum_{u=1}^{N} x_{iu}^4 - 3\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = -4p^4 \quad (14)$$

3.0 RESULTS

3.1 Construction of Forty-Two Points Second Order Rotatable Design in Three Dimensions

The forty two points is obtained by combining two sets i.e the set denoted by 6s containing twenty four points given in (8) ,with another set of twelve points denoted by S(P,P,o) given in (13)assuming s in (8) to be 4.

The sums of squares for the combined sets up to the power of four gives

$\sum_{u=1}^{42} x_{iu}^2$= 2s (r^2+b^2) + 8p^2 = 42λ_2

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$\sum_{u=1}^{42} x_{iu}^{4} = \frac{1}{2}S\,(3r^{4} + 4b^{4}) + 8p4 = 126\lambda_{4}$ (15)

$\sum_{u=1}^{42} x_{iu}^{2}\, x_{ju}^{2} = S\, r^{2}(r^{2}+8b^{2}) + 4p^{4} = 42\lambda_{4}$

And all other odd order powers are zero

The set of forty-two points denoted by R_2 is given by;

$R_2 = s\,(p, p, 0) + 6s$

the excess function for T(r, o, b) given in (4) is added to the excess function for S(p, p, o) given in (14) assuming the value of S to be 5 to give;

$\sum_{u=1}^{42} x_{iu}^{4} = 3\sum_{u=1}^{42} x_{iu}^{2}\, x_{ju}^{2} = 3r^{4} - 24r^{2}b^{2} + 8b^{4} - 3.2p^{4} = 0$ (16)

Equation (16) is solved to obtain the values of r and b in terms of p as follows;

Let $r^{2} = xp^{2}$ and $b^{2} = yp^{2}$ (17)

Substituting (17) to (16) gives

$3x^{2} - 24xy + 8y^{2} - 3.2 = 0$ (18)

Making x the subject of the formula in (18) results to;

$x = \frac{24y \pm \sqrt{480y^{2}+38.4}}{6}$.

The values of y which together with the corresponding values of x makes real and positive values are;

$y \geq 0.632455532$,

Taking an arbitrary value of y to be 1, the corresponding value of x is o.2052668078,

Thus $x = 0.2052668078$ (19)

And $y = 1$

Substituting (19) to (17) gives

$r^{2} = 0.2052668078p^{2}$ (19)

$b^{2} = p^{2}$

Substituting (19) to (15) for the value of $s = 5$ gives,

$\lambda_2 = 0.47744478p^{2}$ $\lambda_4 = 0.1453651466p^{4}$, (20)

Substituting (20) to the non-singularity conditions given in (5) for k=3 factors proves that the condition in (5) is satisfied thus the 42 points forms a second order rotatable design in three dimensions.

4.0 APPLICATIONS

The forty-two points considered provides spherical information contours besides optimum combination of treatments in industrial, medical and agricultural fields which results in economic use of scarce resources in the relevant production processes.

It is important to note that the practical applications of these methods are not automatic, sometimes judgement is required if an experimenter applies insufficient intellect to his results, he is likely to suffer as in any other method of experiment and it is likely to happen in new experiments where unfortunate selection of experimental units is made.

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The second order rotatable design will be readily available when an experimenter wants to carry out experiment where the responses will facilitate the estimation of the linear and quadratic co-efficient.

5.0 CONCLUSIONS AND RECOMMENDATIONS

The response surface methodology in this case is used to approximate the functional relationship between the performance characteristics and the design variables. After experimentation, the resulting response is used to construct response surface approximation model using least squares regression analysis. This study recommends the extension of this class of designs to more than three factors to cater for experiments where more than three factors are required.

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