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Approximating Analysis of the Dzektser Mathematical Model

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ABSTRACT

The article studies the solvability of the Dzektser mathematical model according to the theoretical consequences in the quasi-Sobolev. It employs the results to improve an algorithm form of the approximate approach to find approximate solutions of the Dzektser model. The work investigates the analog of the Dirichlet problem in a bounded domain with boundary conditions for the Dzektser model. A suggested numerical method allows us to find approximate solutions of the Dzektser mathematical model under consideration in quasi-Sobolev space. The convergence of the approximate solution to the analytical solution is fulfilled.

Key Words: Dzektser mathematical model, Phase space, Quasi-Sobolev space.

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1. INTRODUCTION

The first release of the evolution models of Sobolev type was in the different works in the nineteenth century however an organized study of the evolution models started in the second half of the last century with the works of S. L. Sobolev and A. Poincare [1]. Interestingly the study of the evolution models of the Sobolev type for the last time has increased virtually [2], there was a requirement to study it in quasi-Banach space. Moreover, the need is dictated not so much by the desire to expand the theory, but by the desire to comprehend non-classical models of mathematical physics [3] in a quasi-Banach space [4]. Notice that the models of the Sobolev type are called evolution if its solution exists only on the half-axis R_+ [5].

First appearing of holomorphic degenerate semigroups of operators as a solution semigroups of linear evolution equations of Sobolev type [6],

$$L\dot{u}=Mu, \qquad (1)$$

such that the operator $L \in L(U;F)$ is a linear and bounded , the operator $M \in Cl(U;F)$ is a linear, closed and densely defined, and F - Banach space. set out a complete theory for semigroups, theory of semigroups extended to the Frechet space [7,8].

A solution u = u(t) of the equation (1) is called a solution of the Cauchy problem

$$u(0) = u_0, \qquad (2)$$

Quasi-Banach spaces and bounded and closed linear operators defined on them and quasi-Sobolev spaces and powers of Laplace quasi-operator. are considered in this work.

2. THEORETICAL ANALYSIS AND SOLVABILITY OF THE DZEKTSER MATHEMATICAL MODEL

Consider a weakened (in the sense of S.G. Crane) Showalter-Sidorov problem

$$\lim_{t\to 0+} P(u(t)-u_0) = 0 \qquad (3)$$

where P is aprojector, for a nonhomogeneous linear evolution equation of the Sobolev type

$$L\dot{u} = Mu + f, \qquad (4)$$

where a vector function $f:[0,\tau]\to U$, $f = f^0 + f^1$, $f^1 = Qf$, $f^0 = f - f^1$, will defined below, $\tau \in R_+$.

Theorem 1. For any vector function $f = f(t)$ such that $f^0 \in C^1((0,\tau);F^0)$, $f^1 \in C((0,\tau);F^1)$ and for any vector $u_0 \in U$ there exists a unique solution $u \in C^1((0,\tau);U)$ for the equation (4) with the condition (3), which have the form

$$u(t) = -M_0^{-1} f^0(t) + U^t u_0 + \int_0^\tau U^{t-s} L_1^{-1} f^1(s)\, ds. \qquad (5)$$

Proof. Indeed, the fact that $u = u(t)$ satisfies the equation (4) with the condition (3), establishing by direct verification.

Now considering the equation Dzektser

$$(\lambda - \Lambda)u_t = (\alpha\Lambda^2 + \beta\Lambda)u + f, \quad \lambda, \beta \in R, \ \alpha \in R_+ \qquad (6)$$

in the quasi Sobolev spaces $U = \ell_q^{m+2}$ and $F = \ell_q^m$, $m \in \mathrm{R}$, $q \in R_+$. The domain is defined by $\mathrm{d}om(\alpha\Lambda^2 + \beta\Lambda) = \ell_q^{m+4}$. It holds By the theorem 1.

Corollary 1. For any $m, \lambda, \beta \in R$, $\tau, q, \alpha \in R_+$, $u_0 \in U$, $f^0 \in C^1((0,\tau);F^0)$ and $f^1 \in C((0,\tau);F^1)$ there exists a unique solution $u \in C^1((0,\tau);U)$ for the problem (3), (6), which have the form

$$u(t) = -M_0^{-1} f^0(t) + U^t u_0 + \int_0^\tau U^{t-s} L_1^{-1} f^1(s)\, ds.$$

Here

$$F^0 = \begin{cases} \{0\}, & \text{если } \lambda_k \neq \lambda \text{ при всех } k \in N; \\ \{ f \in F : f_k = 0, \ k \in N \setminus \{\ell : \lambda_\ell = \lambda\}\}; \end{cases}$$

$$F^1 = \begin{cases} F, & \text{если } \lambda_k \neq \lambda \text{ при всех } k \in N; \\ \{ f \in F : f_k = 0, \lambda_k = \lambda\}; \end{cases}$$

$$M_0^{-1} = \begin{cases} \mathrm{O}, & \text{если } \lambda_k \neq \lambda \text{ при всех } k \in N; \\ \sum\limits_{k \in N : \lambda_k = \lambda} (\alpha\lambda_k^2 + \beta\lambda_k)^{-1} e_k. \end{cases}$$

$$U^t = \begin{cases} \sum\limits_{k=1}^{\infty} e^{\mu_k t} \langle \cdot, e_k \rangle e_k, & \text{если } \lambda_k \neq \lambda \text{ при всех } k \in N; \\ \sum\limits_{k \in N : k \neq \ell} e^{\mu_k t} \langle \cdot, e_k \rangle e_k, & \text{существует } \ell \in N : \lambda_\ell = \lambda. \end{cases},$$

where $\mu_k = (\alpha\lambda_k^2 + \beta\lambda_k)(\lambda - \lambda_k)^{-1}$.

$$L_1^{-1} = \begin{cases} \sum\limits_{k=1}^{\infty} (\lambda - \lambda_k)^{-1} \langle \cdot, e_k \rangle e_k, & \text{если } \lambda_k \neq \lambda \text{ при всех } k \in N; \\ \sum\limits_{k \in N : k \neq \ell} (\lambda - \lambda_k)^{-1} \langle \cdot, e_k \rangle e_k, & \text{существует } \ell \in N : \lambda_\ell = \lambda. \end{cases}$$

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3. APPROXIMATING ANALYSIS OF THE DZEKTSER MATHEMATICAL MODEL

An approximate solution of the problem (1)-(2) based on the projection method which is modified because the problem may be degenerate. A brief description of the essence of the numerical method to find an approximate solution $\tilde{u}(t)$ by using:

$$\tilde{u}(t) = uN\ (t) = \sum_{k=1}^{N} u_k(t)e^k \qquad (7)$$

where $N \in$ N.

It is necessary when we apply the projection method to take in account firstly, the effects of degeneracy of the equation and, secondly fulfillment of required accuracy ϵ. To select a number , first of all defined a required estimation by

$$\int (\boldsymbol{r}\|u(t) - \tilde{u}(t)\|)dt < \epsilon,\ for\ given\ \epsilon \qquad (8)$$

We consider the following two cases:

Case one: $P_n(x_k)\ f= 0$, $k = 1, ..., N$, in this case, all the equations of the system will be first-order ordinary differential equations, by solving a system, we get the unknown functional coefficients $u_k(t)$, $k = 1, ..., N$ in the approximate solution $\tilde{u}(t)=u^N(x,t)$.

Case two: $P_n(x_{k_j}) = 0$, for some kj, in this case the equations of the system with the number kj will be algebraic equations and the rest of the equations will be differential equations.

The steps of the algorithm finding an approximate solution for the problem (1)-(2) are :

i- finding the number N , we can find the approximate solution depending on the number N ;

ii- checking mathematical model according to the given parameters to which two cases (mentioned above) it refers;

iii- calculation of the approximate solution for a given initial sequence by using a modified projection method.

4. EXPERIMENTAL EXAMPLES OF THE DZEKTSER MATHEMATICAL MODEL

Consider the Dzektser model

$$(\lambda - \Lambda)u_t = (\alpha\Lambda^2 + \beta\Lambda)u + f, \quad \lambda, \beta \in R,\ \alpha \in R_+ \qquad (9)$$

$$u(x,0) = u_0(0), \quad x \in [0,l] \qquad (10)$$

$$u(0,t) = u(l,t) = 0, \quad t \in [0,T] \qquad (11)$$

in quasi-Sobolev spaces U = A^{r+2n} and F = A^r, , such that $r \in$ R, $q \in$R$_+$, $L = P_1(\Lambda) = \lambda + \Lambda$ and $M = Q_0(\Lambda) = \alpha$I, the operators $L, M \in$ L(U; F).

Example 1.

To find a numerical solution of the mathematical model (9)-(11), construct the polynomials from the Laplace quasi-operator.

Let $T = 6, m = 2, q = 5$, $l = 2\pi$, u_{0k} , and $\lambda_k = k$.

The degeneracy of the mathematical model and apply the phase space method, find the number of nonzero terms of the approximate solution $\tilde{u}(t)$ which are necessary to fulfil a given accuracy $\epsilon = 0.01$, and the approximate solution is shown in Figure 1.

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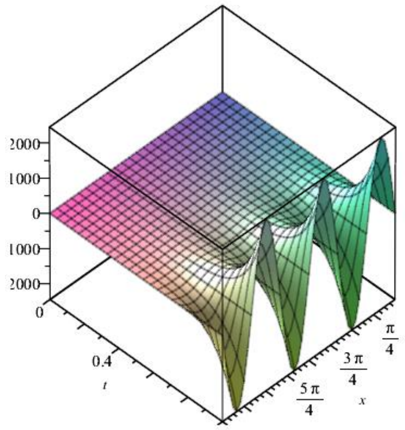

Fig. 1 Approximate solution of example 1

Example 2.

To find a numerical solution of the mathematical model (9)-(11), construct the polynomials from the Laplace quasi-operator.

Let $T = 10, m = 2, q = 7$, $l = 2\pi$, u_{0k} , and $\lambda_k = k$.

The degeneracy of the mathematical model and apply the phase space method, find the number of nonzero terms of the approximate solution $\tilde{u}(t)$ which are necessary to fulfil a given accuracy $\epsilon = 0.001$, and the approximate solution is presented in Figure 2.

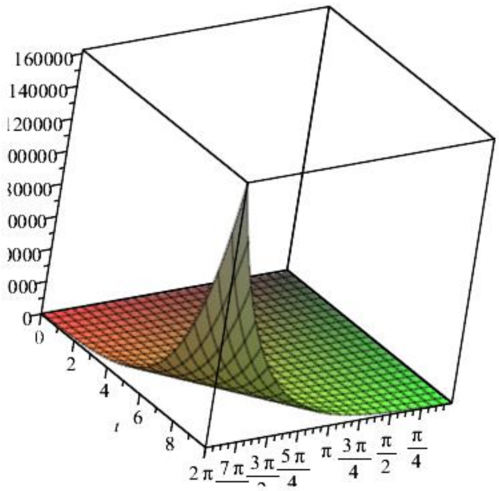

Fig. 2 Approximate solution of example 2

5. CONCLUSIONS

The numerical method of the approximate solution of Dzektser mathematical model in the framework of the Cauchy problem n quasi-Sobolev space Sufficient conditions for the existence of invariant space of solutions. The enhanced algorithm applied a numerical method for studying Dzektser mathematical model in quasi-Sobolev space. A program implements an algorithm of a numerical method for studying of the Dzektser mathematical model in quasi-Sobolev space. A suggested numerical method allows us to find approximate solutions of the Dzektser mathematical model under consideration in quasi-Sobolev space.

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