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Measurement of Temperature Distributions on a Fin Surface by Analytical, Experimental and Numerical Methods

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ABSTRACT

Measurement of temperatures on a fin surface along with calculation of heat transfer rate is a very common experiment generally performed in Heat and Mass Transfer Laboratory. To make things interesting we analysed the results from this experimental study and compared them with theoretical (analytical) and numerical results. The theoretical study is done by standard fin formulation and the numerical analysis is performed by discretizing the governing equation of fin in algebraic mode of equations. For both of these cases we assumed suitable boundary conditions. It has been observed that the trend of temperature distribution in three cases is similar although errors are associated with experimental observations and numerical study. If we able to minimize these errors then we can properly predict the analytical results both from experimental and numerical studies.

Key words: *Heat and Mass Transfer, Fins, Newtons law.*

1. INTRODUCTION

A fin is an extended surface which is attached to any hot object in order to increase the rate of the heat transfer from the object to the environment by decreasing the resistance to convective heat transfer. This is governed by Newton's law of cooling which is given by $Q = \square A(T_s - T_\infty)$, where, Q is the rate of heat transfer, T_s, T_∞ are the temperatures of the surface of the body and surrounding respectively, h is the convective heat transfer coefficient. Therefore rate of heat transfer can be enhanced by increasing h and A or by reducing T_∞ which is practically not possible. To increase the value of h needs forced convection with several external aids which not only increase the installation cost, but also increases the running cost. Therefore fins are introduced which increase Q by increasing the surface area of the desired hot body.

Fins are most commonly used for heat exchange such as radiator in car, CPU, heat sink and heat exchanger in power plant, cooling coils and condenser coils in refrigerators and air conditioners, small capacity compressor, electric motor bodies, transformer and electric components etc.

To evaluate the performance of a fin, both the experiment and numerical techniques along with analytical studies have been conducted by considering a pin fin which is readily available in the Heat and Mass transfer laboratory of the authors Institute. All these three methods in detail are discussed in subsequent sections to understand the assumptions considered for these studies. Finally, the obtained results by both the experimental and numerical approaches are compared with the analytical solution and conclusion has been drawn.

2. EXPERIMENTAL PROCEDURE

The experimental setup is available in Heat & Mass Transfer Laboratory in authors' Institute which is used to perform experiments on both the natural (free) and forced convection. In the present study we calculated the data related with free-convection only. This setup is shown in Fig. 1, where the fin is installed inside the rectangular duct normal to it. Both its base and tip are attached to the duct. The inside view of the duct is given in Fig. 2 to understand the proper position of the fin. A heater is attached outside the duct such that it heats up one end of the fin (base) and heat flows to another end (tip). Heat input to the heater is given through variac (variable transformers), whereas digital voltmeter and digital ammeter are provided to measure heat. Digital temperature indicator measures the temperature distribution along the fin with the help of temperature sensors which are installed at different locations of the fin periphery. The length and cross-sectional area of the fin are respectively 0.145m and 0.015 m^2.The distance of first temperature sensor (T_1) from the base end is 0.03m and distance of the rest temperature sensors T_2, T_3, T_4, T_5 are 0.055m, 0.080m, 0.105m and 0.130m respectively from the same end. The temperature sensor T_6 is placed at base which is used to calculate the heat transfer rate by fin considering pure conduction mode of heat transfer between T_6 and T_7, where T_7 is placed at a distance of 0.01 m from T_6. For experimental and analytical study T_6 value is made equal to the base temperature T_0. The surrounding ambient temperature (T_∞) is measured by a temperature sensor T_8 which is installed at the duct surface. It should be noted that the equipment needs a good amount of time (approximately 2 hours) to reach steady state after which the temperatures are measured. The measured values are then tabulated in $\frac{x}{l}$ vs $\frac{\theta}{\theta_0}$ form. Here, x is the position of the temperature sensors measured from the base, l is the length of the fin, $\theta = T - T_\infty$ and $\theta_0 = T_0 - T_\infty$.

Figure 1: Heat transfer experimental setup for a pin fin

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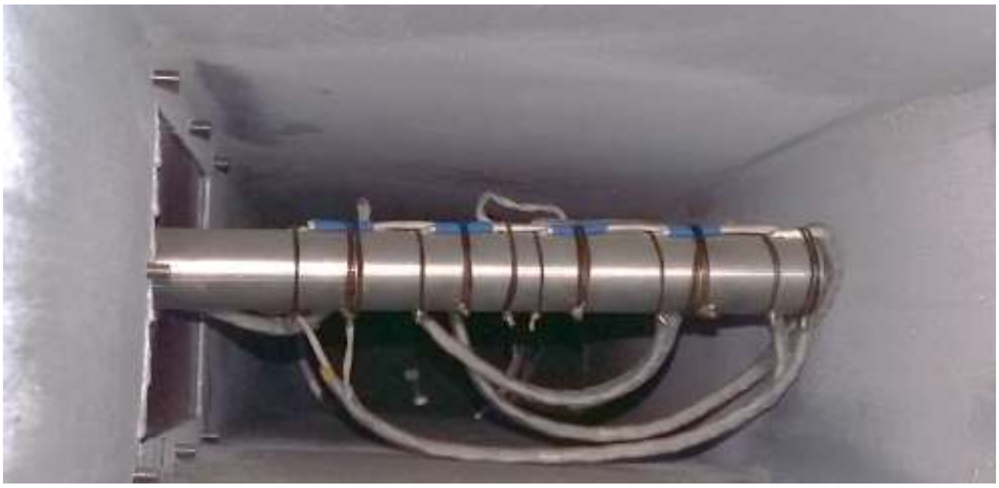

Figure 2: **The position of the pin fin inside the duct**

3. THEORETICAL FORMULATION OR ANALYTICAL STUDY

Consider a small area of *dx* length on a uniform cross-sectional fin as shown in Fig. 3. The energy balance can be done by using law of conservation of energy:

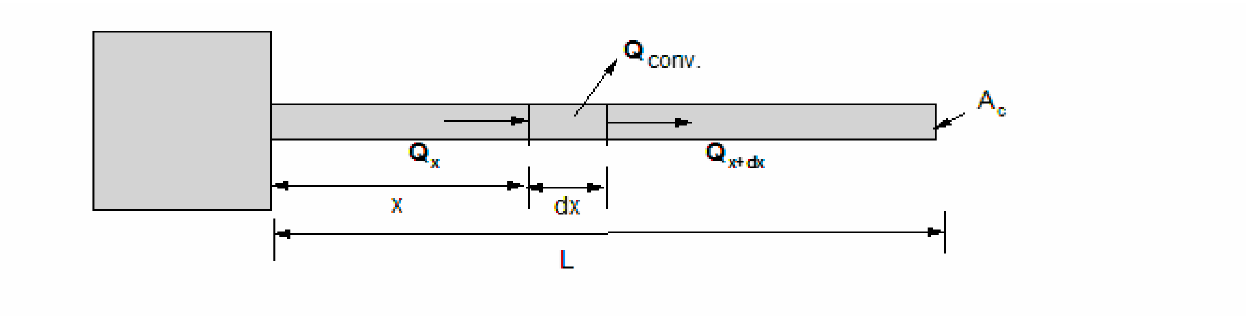

Figure 3: **Schematic diagram of fin with conduction and convection mode of heat transfer**

$$\dot{Q}(x + dx) = \dot{Q}(x) + d\dot{Q}_{convection} \quad (3.1)$$

From Fourier's law of heat conduction, we obtain

$$\dot{Q}(x) = -kA_c\left(\frac{dT}{dx}\right) \quad (3.2)$$

where, A_c is the cross-sectional area of the differential element, k is thermal conductivity of the material.

Furthermore, the convective heat flux can be determined by Newton's law of cooling.

$$Q = h(T - T_\infty) \quad (3.3)$$

Here, T_∞ is the temperature of the surrounding; h is convective heat transfer coefficient.

Now, by using equations (3.1) – (3.3), the following differential equation for the temperature is simplified to equation (3.4) as

$\frac{d^2T}{dx^2} = \frac{\square P}{kA_c}(T - T_\infty)$ (3.4)

The solution of the equation (3.4) is

$\Theta(x) = C_1\, e^{mx} + C_2\, e^{-mx}$ (3.5)

where, $m = \frac{\square P}{KA_c}$, C_1 and C_2 are arbitrary constants which can be found out using different boundary conditions. The following cases may be considered:

Case 1: The fin is infinitely long and the temperature at the end of the fin is same to surrounding fluid.

Case 2: The end of the fin is insulated.

Case 3: The fin is of finite length and losses heat by convection from its tip.

The analytical calculations of steady state heat transfer for a fin of the finite length losing heat at the tip can be obtained from any standard Heat Transfer book [1, 2]. Here, the boundary conditions are:

(i). at $x = 0$, $\Theta = \Theta_0$ and

(ii) at $x = l$, heat conduction from the tip of the fin is equal to the heat convection from the same surface, i.e., -

$kA_c\left[\frac{dt}{dx}\right]_{x=l} = hA_s(t - t_0)$ (3.6)

Here, A_c and A_s both represent the area of the tip surface.

Applying the boundary condition (i), we get

$C_1 + C_2 = \Theta_0$

Solving these values of the constant C_1 and C_2 in above equation, we obtain non-dimensional temperature as,

$\frac{\Theta}{\Theta_0} = \frac{\cos\square(m(l-x)) + \frac{\square}{km}(\sinh(m(l-x))}{\cosh(ml) + \frac{\square}{km}(siin\square(ml))}$ (3.7)

and heat transfer rate by fin as,

$Q = \sqrt{\square PkA_c}\,(t_0 - t_a)\left[\frac{\tanh(ml) + \frac{\square}{km}}{1 + \frac{\square}{km}\tanh(ml)}\right]$ (3.8)

Using equation (3.7) the measured temperatures are then tabulated in $\frac{x}{l}$ vs $\frac{\theta}{\theta_0}$ form.

4. NUMERICAL METHOD

Numerical method is also applied for the same problem. This method is used by transforming the partial differential equations into approximate algebraic difference equations known as discretization. This can be done effectively by three different methods, namely,

a. Finite difference method
b. Finite volume method
c. Finite element method

Here, finite difference method is used. It should be mentioned here that numerical method is always associated with some errors [3]. The first kind of error generates due to discretization of partial derivatives into algebraic set of equations known as discritization error. Apart from this a second kind of error also generates due to rounding off the numbers to some significant figure in repeating calculations called round-off error. As the present case is considered in steady state (time independent), therefore the numerical result is free from round-off error.

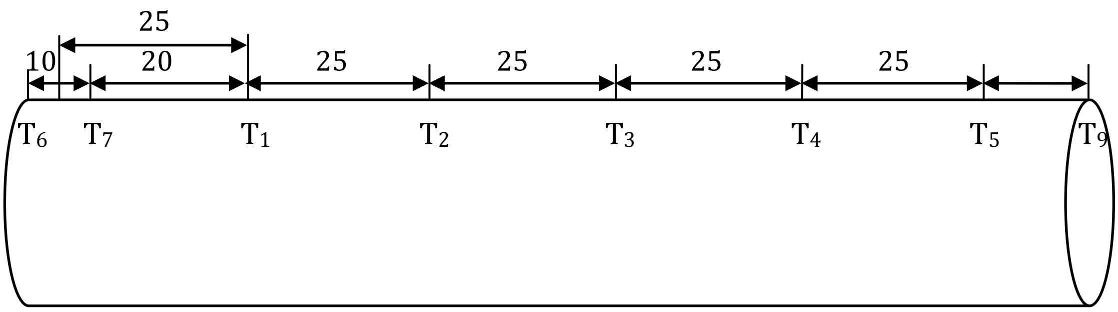

Figure 4: **Pin fin with temperature sensors location in case of numerical study. All dimensions are in mm.**

In the present simulation, we consider one-dimensional (along the length of fin) heat transfer and have selected 7 grid points for this study (points 1, 2, 3, 4, 5 and 9 are the six points as per Fig. 4). The first grid point is placed at the middle of point 6 and 7 (refer Fig. 4) and temperature of this point is known (T_0). Temperature at point 9 (T_9) is calculated using the boundary condition for fin losing heat at tip. To calculate the temperature of the intermediate grid points, a second-order accurate central difference scheme is used explicitly considering its higher-order of accuracy as compared to first-order discretization. As the number of grid points is few (seven in number), discritization with higher order accuracy was not possible in the present case.

The algebraic set of equations for intermediate grid points obtained from steady state 1-D Fourier law of heat conduction are as follow:

$T_0 - 2T_1 + T_2 = 0$

$T_1 - 2T_2 + T_3 = 0$

$T_2 - 2T_3 + T_4 = 0$

$T_3 - 2T_4 + T_5 = 0$

$\beta T_4 - \gamma T_5 + \delta T_9 = 0$

Here, β, γ and δ are given as $\beta = \frac{\Delta x_{4-5}}{\alpha}$, $\gamma = \frac{\Delta x_{4-5}+\Delta x_{5-9}}{\alpha}$ and $\delta = \frac{\Delta x_{5-9}}{\alpha}$, such that $\alpha = \frac{\Delta x_{i-1}\Delta x_i}{2}(\Delta x_{i-1} + \Delta x_i)$. Δx_i represents the axial distance between points i and i+1. Putting these values finally we obtained for this pin fin case, α = 7.5 × 10^6, β = 3333, γ = 5333 and δ = 2000. A set of 5 equations from discritization are finally obtained, which we solved by using Gauss- Seidal numerical technique [4].

5. RESULTS AND DISCUSSION

In the present analysis three different input voltage of power supply are considered. They are 50V, 70V and 90V respectively. The base temperature of the fin is directly proportional to the input voltage. Therefore we obtain three different temperature distributions for three different power supplies.

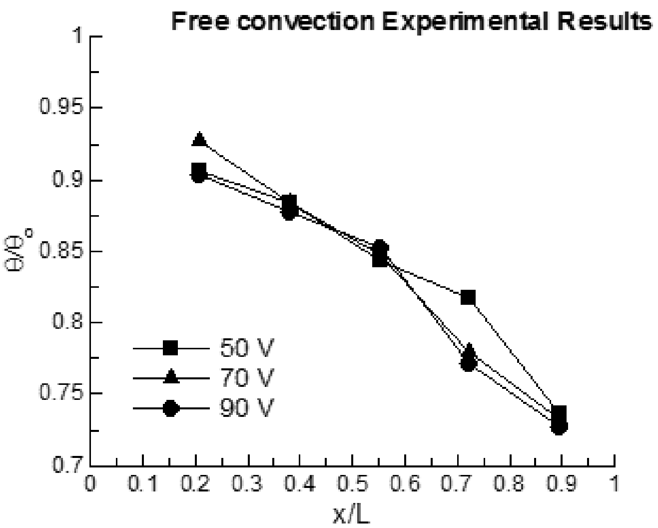

Figure 5: Non-dimensional temperature distribution with non-dimensional fin length in experimental analysis for three different power supplies (voltage).

Figure 5 illustrates the non-dimensional temperature distribution (θ/θ_0) with non-dimensional fin length (*x*/*l*) for three different voltages (50V, 70V and 90V) in experimental analysis. We observed that although the non-dimensional results are same in case of higher voltage (70V and 90V); there is fluctuation in the temperature profile of 50V. This may be attributed to the experimental errors associated with the variac used at low voltage. The analytical result in Fig. 6 and numerical result in Fig. 7 also show the similar trend. This is because in both these cases the base temperature and the surrounding temperature are obtained from the experimental analysis. It has also been observed that the fin temperatures vary linearly in case of numerical analysis, where as for analytical study the variation is of parabolic nature. The linear form of discretized equations used in numerical solution is the reason behind this.

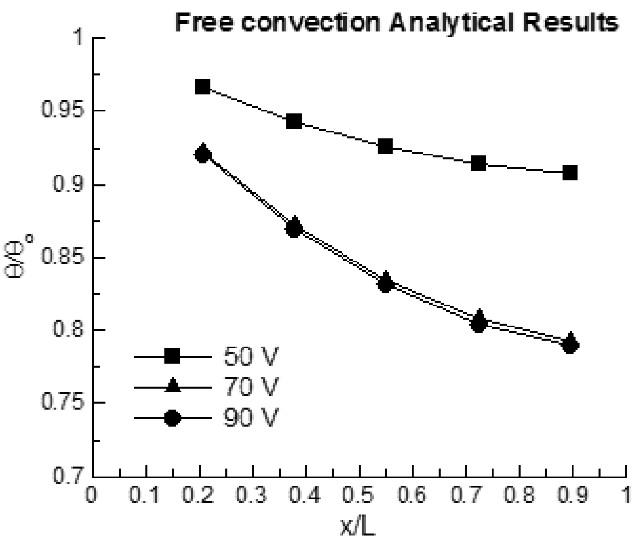

Figure 6: Non-dimensional temperature distribution with non-dimensional fin length in analytical study for three different power supplies (voltage).

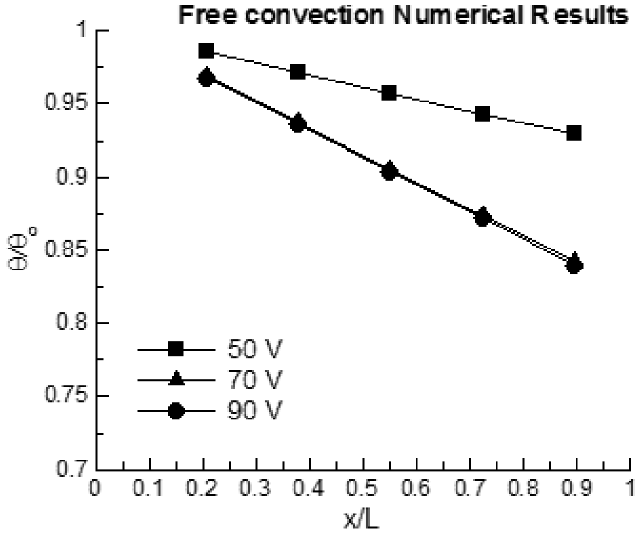

Figure 7: **Non-dimensional temperature distribution with non-dimensional fin length in numerical method for three different power supplies (voltage).**

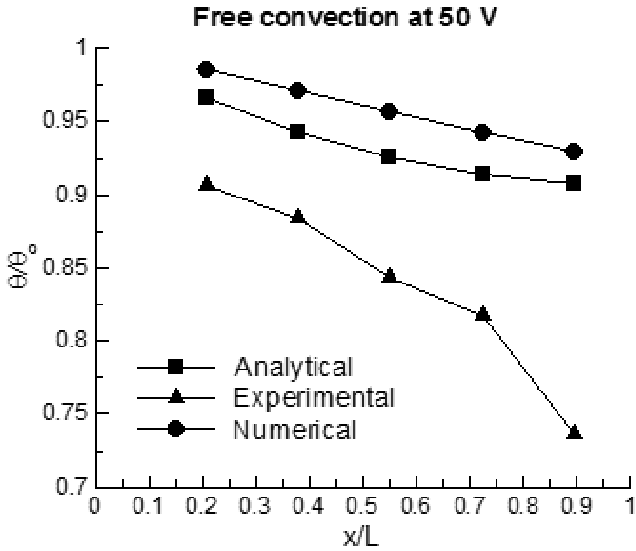

Figure 8: **Non-dimensional temperature distribution with non-dimensional fin length for three different methods at 50V.**

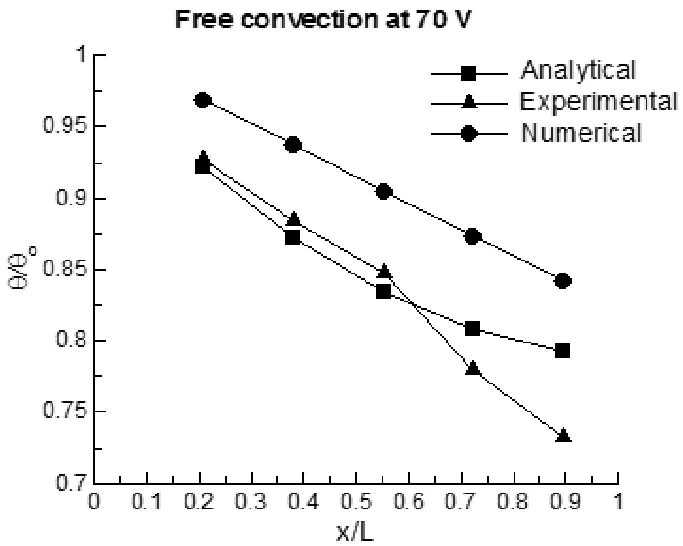

Figure 9: **Non-dimensional temperature distribution with non-dimensional fin length for three different methods at 70V.**

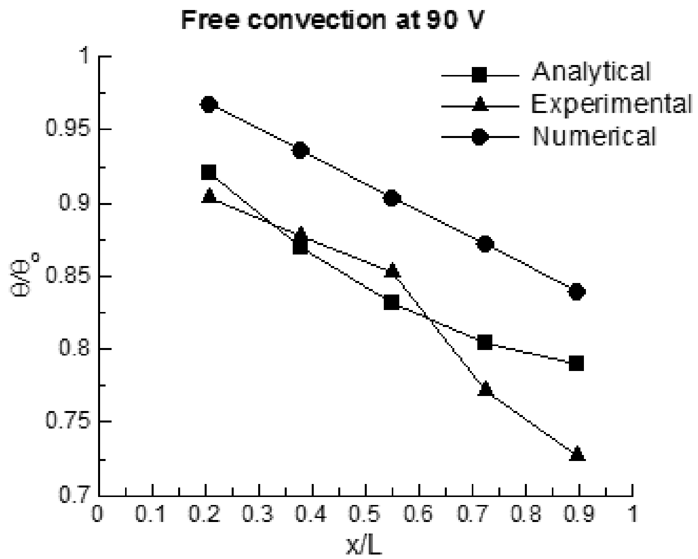

Figure 10: **Non-dimensional temperature distribution with non-dimensional fin length for three different methods at 90V.**

To compare the results from different methods Figs. 8-10 are drawn at three different voltages. The results show that the experimental study in most cases under-predict the theoretical results and the numerical analysis over-predict the temperature values. At low power supply (50V), the analytical and numerical results are of same trend and close to each other. But as power supply increases (70V and 90V), the experimental results come close to analytical results. This proves at high voltage to get more accurate numerical result it is important to use higher-order discretization scheme which will take care of the discretization error.

CONCLUSIONS

Experimental, analytical and numerical studies have been made on a pin fin in free convection. The fin was placed in a duct to perform heat transfer study. For analytical and numerical approach suitable boundary conditions have been considered to properly mimic the experimental observations. The following conclusions have been made:

(a) The experimental results at low power supply (50V) are not satisfactory which can be attributed to error associated with voltage measurement (variac).

(b) At high power supply (70V, 90V) the non-dimensional temperature distribution are almost same pointing towards errorless calculations at these high voltages.

(c) Similar trend is also observed in analytical and numerical study. This can be attributed to the base and surrounding temperature values which we taken directly from experimental results.

(d) The analytical temperature distributions are of parabolic form due to quadratic nature of the governing equation.

(e) For numerical analysis, the temperature distributions follow linear trend. This is directly related to the second – order discretization of the governing equation which makes the discretized equation linear.

(f) The experimental results under-predict the analytical results, whereas the numerical results over-predict the analytical results.

(g) At low power supply (50V), numerical results predict the analytical results reasonably well as compare to experimental results indicating problem with variac used in experimental study at low voltage.

(h) At high power supply (70V, 90V), experimental results are closer to the analytical results. This indicates a higher-order discretization scheme is required in numerical simulation to obtain better results at these high voltages.

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