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THE FINITE DIFFERENCE EQUATIONS OF SUCCESSIVE APPROXIMATION METHOD FOR THE CALCULATION OF BENDING BEAMS OF VARIABLE THICKNESS UNDER THE STATIC ACTION OF LOADS

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ABSTRACT

The finite difference equations of the successive approximation method (SAM) which substitute the differential equations of bending beam of a variable stiffness are obtained. Difference equations of SAM, which approximate the limit conditions of the hand ends of the beam, are also obtained: simply supported hand end; rigidly fixed hand end and free hand end. On the basis of the obtained equations, a numerical algorithm was developed for calculating beams of constant and variable thickness under the action of various static loads. According to this algorithm, a program for calculating beams on a computer was performed. Variable stiffness simple supported hand ends beams, rigidly fixed hand ends beams with uniformly distributed loads along their lengths, with concentrated force, were calculated. A cantilever beam of variable thickness was also calculated under the action of the uniformly distributed load over its entire length. The examples presented here show the accuracy of the results and the simplicity of the algorithm. Checks for integral equilibrium conditions of beams were performed to validate the newly obtained results.

Key words

Successive Approximation Method, Beam of Variable Thickness, Numerical Algorithm, Equilibrium Conditions, validate.

1. INTRODUCTION

The foundation beams, the columns of some buildings, the pillars of electrical transportation and the chimneys are generally beams of constant and variable stiffness. During the process of their exploitation, these structures undergo the action not only of static charges, but also dynamic loads. The calculation of such structures must be accurate and easy to execute. Despite the practical importance of such elements found in literature involved, many questions related to their calculation are still relevant. In fact works of researchers address the problem of beams see [1], [2], [3], [4], [5]. The calculation of these structures requires the implementation of tools for modeling mechanical behavior increasingly sophisticated, and taking into consideration the specificities of these structures. Their calculation by the analytical methods remains very tedious and bulky see [1], [6]. The numerical methods see [2], [3], [7], [8], [10] are more efficient. Among numerical methods, the finite element method is the most used, but it presents a number of difficulties such as the formation of the stiffness matrix and the tightening of the mesh around the specific zones. New numerical methods that are much more resonant, simple and that yield appreciable results are developed by other researchers [2], [11], [12]. Among these methods we have the Successive Approximation Method (SAM) which is the subject of this article. More precisely, we use this Method to develop an algorithm for calculating a bending beam of variable stiffness.

The work is organized as follows. Section 2 describes the equations of the model problem and the limit conditions. In section 3 we proceed with the methodology of the implementation of the Successive Approximation Method which is divided into three points:

- Introduction of the new dimensionless parameters in the system of equations thus obtained and in the equations describing the boundary conditions;
- Substitution of the new differential equations by the Successive Approximation Method, which permit to obtain a system of algebraic equations;
- Elaboration of a calculation algorithm;

Section 4 is devoted to the validation of our approach (Successive Approximation Method) through the numerical resolution of the test problems.

2. EQUATIONS OF THE PROBLEM'S MODEL AND THE LIMIT CONDITIONS

2.1 Equation of the Problem's Model

The differential equation of the bending beam of variable stiffness, written within the framework of known assumptions, will be obtained from (23) in [1] with $k = 0$, where k- is the coefficient takes into consideration the reactive effects of the base. We write this equation with a replacement y, x respectively W, Z :

$$\frac{d^2}{dZ^2}\left[EJ(Z).\frac{d^2W(Z)}{dZ^2}\right] = q(Z) \qquad (1)$$

where E is modulus of elasticity; $J(Z)$ is the moment of inertia of an arbitrary section of the beam; $W(Z)$ is vertical movement; $q(Z) -$ intensity of the transverse load distributed according to an arbitrary law. Equation (1) can be replaced by two differential equations of the second order. To do this, we denote the expression in square brackets through $-M(Z)$:

$$\frac{d^2M(Z)}{dZ^2} = -q(Z) \qquad (2)$$

$$\frac{d^2W(Z)}{dZ^2} = -\frac{M(Z)}{EJ(Z)} \qquad (3)$$

where $M -$ is the bending moment.

Numerical integration of (2), (3) is performed under the following boundary conditions:

2.2 Limit Conditions

2.2.1 Simply Supported Left-hand End of the Beam

Let's consider the beam Figure1 be simply supported at its left-hand end (at nod 0).

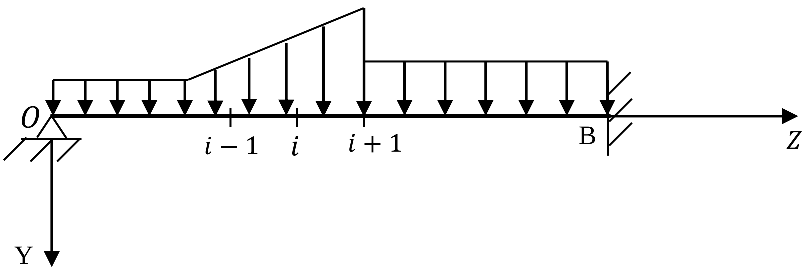

Figure 1: Beam simply supported at the left-hand end and rigidly fixed at the other hand end under the action of an arbitrary distributed load

For this nod we have: a) $W(Z) = W_0$ b) $M(Z) = M_0$ (4)

where W_0, M_0 are given values of the deflection and bending moment of the beam at nod 0, respectively.

2.2.2 Rigidly Fixed Left-hand End

If the beam is rigidly fixed at its left-hand end, then for this end the following equalities are valid:

a) $W(Z) = W_0$ b) $\frac{dW}{dZ} = W_0^Z$ (5)

where $W_0^Z -$ is the specified angle of rotation.

2.2.3 Free Left-hand End

For the free left-hand end of the cantilever, the limit conditions will be written as follow:

a) $M(Z) = M_0$ b) $\frac{\partial M}{\partial Z} = M_0^Z$ (6)

where M_0^Z is the specified shear force.

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3 METODOLOGY OF THE IMPLIMENTATION OF THE SUCCESSIVE APPROXIMATION METHOD (SAM)

3.1 Introduction of New Dimensionless Parameters

3.1.1 System of Partial Differential Equations

Introducing the dimensionless parameter $\xi = \frac{Z}{l}$, from (2), (3) we get:

$$\frac{\partial^2 m}{\partial \xi^2} = -p \quad (7)$$

$$\frac{\partial^2 v}{\partial \xi^2} = -m\gamma \quad (8)$$

where $m = \frac{M(Z)}{q_{0l^2}}$ $v = \frac{EJ_0 W(Z)}{q_{0l^4}}$ $\gamma = \frac{EJ_0}{EJ(Z)}$ p= $\frac{q(Z)}{q_0}$ (8a)

l characteristic of the beam size; q_0 the intensity of the load at any point; EJ_0 is fixed beam stiffness.

3.1.2 Limit Conditions

3.1.2.1 Simply Supported Left-hand End

Let's write (4) in dimensionless values for the nod i of the simply supported hand end of the beam while $\xi = 0$:

a) $v_i = v_0$; b) $m_i = m_0$, (9)

where $i = const$; v_0, m_0 given values of deflection and bending moment, respectively; $v_0 = \frac{W_0 EJ_0}{q_0 l^4}$, $m_0 = \frac{M_0}{q_0 l^2}$.

3.1.2.2 Rigidly Fixe Left-hand End

Conditions (5), recorded relative to the dimensionless unknowns, for the nod i of the rigidly fixed hand end of the beam (at $\xi = 0$) can be written as follow:

a) $v_i = v_0$; b) $(\frac{dv}{d\xi})_i = v_i^\xi = v_0^\xi$, (10)

where $i = const$; v_0^ξ is the given rotation angle ; $v_0^\xi = \frac{dv}{d\xi|_{i=0}}$.

3.1.2.3 Free Left-hand End

We write (6) with respect to the dimensionless unknowns for the nod i of free hand end ($\xi = 0$):

a) $m_i = m_0$ b) $\frac{dm}{d\xi} = m_i^\xi = m_0^\xi$ (11)

where $i = const$; m_0; m_0^ξ are the specified values, respectively, of the bending moment and the transverse force at the nod i; $m_0^\xi = \frac{dm}{d\xi|_{i=0}}$.

3.2 Substitution of the Differential Dimensionless Equations and the Limit Conditions by the Finite Difference Equations of Successive Approximation Method (SAM)

3.2.1 Substitution of the Differential Dimensionless Equations

The general equations of a compressed-bent rod of variable stiffness were obtained in [2] on the basis of the difference form of the method of successive approximations. From a comparison (7) with (4.1.1) of [2], it follows that to approximate (7) with the difference equation of SAM on a uniform mesh ($h_{i-1} = h_{i+1} = h$), it suffices to write (4.1.3) of [2] when $k = 0$ ($k = \frac{Nl^2}{EJ_0}$, N where is the compressive force):

$$m_{i-1} - 2m_i + m_{i+1} + \Delta m_i + h\Delta m_i^\xi = -\frac{h^2}{12}\left[{}^d p_{i-1} + 10\,{}^g p_i + {}^g p_{i+1}\right] + \frac{5h^2}{12}\Delta p_i + \frac{h^3}{12}\Delta p_i^\xi \quad (12)$$

where $\Delta m_i = {}^g m_i - {}^d m_i$; $\Delta m_i^\xi = {}^g m_i^\xi - {}^d m_i^\xi$; $m^\xi = \frac{\partial m}{\partial \xi}$; $\Delta p_i = {}^g p_i - {}^d p_i$; $\Delta p_i^\xi = {}^g p_i^\xi - {}^d p_i^\xi$; $p^\xi = \frac{\partial p}{\partial \xi} p^\xi = \frac{dp}{d\xi}$;

$i = 2,3,4, \dots \dots \dots, (n-1)$ is measured along the ξ axis. The upper left indices "g", "d" denote the values of m, m^ξ, p and p^ξ both to the left and to the right of the point i.To approximate (8) with the difference equation of SAM, it suffices to write (12) with replacement of m, p respectively with v, $m\gamma$; for the case of continuous v, γ and continuous derivatives v, this equation will have the form:

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$$v_{i-1}-2v_i+v_{i+1}=-\frac{h^2}{12}\left[\gamma_{i-1}\,{}^{d}m_{i-1}+10\gamma_i\,{}^{g}m_i+\gamma_{i+1}\,{}^{g}m_{i+1}\right]+\frac{5h^2}{12}\Delta(\gamma m)_i+\frac{h^3}{12}\Delta(\gamma m)_i^{\xi} \quad (13)$$

where $\Delta(\gamma m)_i^{\xi}={}^{g}(\gamma m)_i^{\xi}-{}^{d}(\gamma m)_i^{\xi}$; $i=2,3,4,\ldots\ldots\ldots,(n-1)$ is measured along the ξ axis, notice that ${}^{g}(\gamma m)_i^{\xi}={}^{g}\gamma_i^{\xi}\,{}^{g}m_i+{}^{g}\gamma_i\,{}^{g}m_i^{\xi}$, ${}^{d}(\gamma m)_i^{\xi}={}^{d}\gamma_i^{\xi}\,{}^{d}m_i+{}^{d}\gamma_i\,{}^{d}m_i^{\xi}$; therefore, for the case of continuity of γ and derivatives of γ $\Delta(\gamma m)_i^{\xi}=\gamma_i\Delta m_i^{\xi}$, $\Delta(\gamma m)_i=\gamma_i\Delta m_i$. here $\gamma^{\xi}=\frac{d\gamma}{d\xi}$.

Substituting these expressions into (13), we get:

$$v_{i-1}-2v_i+v_{i+1}=-\frac{h^2}{12}\left[\gamma_{i-1}\,{}^{d}m_{i-1}+10\gamma_i\,{}^{g}m_i+\gamma_{i+1}\,{}^{g}m_{i+1}\right]+\frac{5h^2\gamma_i}{12}\Delta m_i+\frac{h^3\gamma_i}{12}\Delta m_i^{\xi} \quad (14)$$

Equations (12), (14) obtained above, together with the boundary conditions, allow us to solve static problems of bending variable-stiffness beams on a load action with an arbitrary distribution law. Below, we consider difference approximations of various variants of boundary conditions.

3.2.2 Limit Conditions

3.2.2.1 Simply Supported Left-hand End

Let's write (9) in dimensionless values for the nod i of the simply supported hand end of the beam while $\xi=0$: a) $v_i=v_0$; b) $m_i=m_0$, (15)

where $i=const$; v_0, m_0 given values of deflection and bending moment, respectively; $v_0=\frac{W_0EJ_0}{q_0l^4}$, $m_0=\frac{M_0}{q_0l^2}$.

If both hand ends of the beam are simply supported, the calculation is reduced to the joint solution of systems of equations similar (12), (14) taking into consideration (15).

3.2.2.2 Rigidly Fixe Left-hand End

Conditions (10), recorded relative to the dimensionless unknowns, for the nod i of the rigidly fixed hand end of the beam (at $\xi=0$) can be written as follow:

$$\text{a) } v_i=v_0; \qquad \text{b) } \left(\frac{dv}{d\xi}\right)_i=v_i^{\xi}=v_0^{\xi}, \quad (16)$$

where $i=const$; v_0^{ξ} is the given rotation angle ; $v_0^{\xi}=\frac{dv}{d\xi|_{i=0}}$.

To approximate (16) b) on a uniform mesh ($h_i=h$), it suffices to write down (4.1.6) of work [2] with replacement w, g respectively by v, γ:

$$v_i^{\xi}=\frac{1}{h}(-v_i+v_{i+1})+\frac{h}{12}\left[5\,{}^{d}(\gamma m)_i+{}^{g}(\gamma m)_{i+1}\right]+\frac{h^2}{12}(\gamma m)_i^{\xi} \quad (17)$$

where $(\gamma m)^{\xi}=\frac{d(\gamma m)}{d\xi}$

Let's write the last member of the right side of (17) in following form:

$$(\gamma m)^{\xi}=\gamma^{\xi}m+\gamma m^{\xi} \quad (18)$$

To approximate m^{ξ} by difference equation of SAM, it suffices to write (18) with replacement v, $m\gamma$ respectively by m, p:

$$m_i^{\xi}=\frac{1}{h}(-m_i+m_{i+1})+\frac{h}{12}\left[5\,{}^{d}p_i+{}^{g}p_{i+1}\right]+\frac{h^2}{12}p_i^{\xi} \quad (19)$$

Substituting (19) into (18), written for nod i , we get:

$$(\gamma m)_i^{\xi}=\left(\gamma_i^{\xi}-\frac{\gamma_i}{h}\right)m_i+\frac{\gamma_i}{h}m_i+\frac{h\gamma_i}{12}\left(5\,{}^{d}p_i+{}^{g}p_{i+1}\right)+\frac{h^2\gamma_i}{12}p_i^{\xi} \quad (20)$$

The final differential expression, approximating (16) b), can be obtain from (17) by substituting into it (20):

$$v_i^{\xi}=\frac{1}{h}(-v_i+v_{i+1})+\frac{h\gamma_i}{12}\left[(4+\frac{h\gamma_i^{\xi}}{\gamma_i})m_i+(1+\frac{\gamma_{i+1}}{\gamma_i})m_{i+1}\right]+\frac{h^3}{12}\left(5\,{}^{d}p_i+{}^{g}p_{i+1}+hp_i^{\xi}\right) \quad (21)$$

Note that instead of (21) you can use (17) and (19) together to solve problems. Equation (21) for the hand end ($\xi=1$) is written in the "mirror image" with the replacement $i+1$ by $i-1$ at the same time, the sign of v^{ξ}, γ^{ξ}, p^{ξ} p^{ξ} is reversed.

Consider another option to approximate (16) b). Express m^{ξ} in (18) through m by a square parabola:

$$m^{\xi}=\frac{1}{2h}(-3m_i+4m_{i+1}-m_{i+2} \quad (22)$$

Substituting (22) into (20), we get:

$$(\gamma m)_i^{\xi}=\left(\gamma_i^{\xi}-\frac{1.5\gamma_i}{h}\right)m_i+\frac{2\gamma_i}{h}m_{i+1}-\frac{0{,}5\gamma_i}{h}m_{i+2} \quad (23)$$

We write (17) without upper left indices, taking into consideration (23):

$$v_i^{\xi}=\frac{1}{h}(-v_i+v_{i+1})+\frac{h}{12}\left[\left(h\gamma_i^{\xi}+3{,}5\gamma_i\right)m_i+(2\gamma_i+\gamma_{i+1})m_{i+1}-0{,}5\gamma_i m_{i+2}\right] \quad (24)$$

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3.2.2.3 Free Left-hand End

We write (11) with respect to the dimensionless unknowns for the nod i of free hand end ($\xi = 0$):

a) $m_i = m_0$ b) $\frac{dm}{d\xi} = m_i^\xi = m_0^\xi$ (25)

where $i = const$; m_0; m_0^ξ are the specified values, respectively, of the bending moment and the transverse force at the nod i; $m_0^\xi = \frac{dm}{d\xi|_{i=0}}$. The difference equation approximating (25) b) is (19).

Equation (19) for the right-hand end ($\xi = 1$) is written in the "mirror image" with the replacement $i + 1$ by $i - 1$, at the same time m^ξ, p^ξ change their sign to the opposite.

3.3 Implementation of an Algorithm of the Calculation

The algorithm for calculating on a regular mesh is as follows. For all the points of the mesh located inside the domain of integration one writes the (12), (14) taking into consideration of boundary conditions. For a beam with two free of support hand ends, these equations are solved simultaneously by considering the limit conditions (15), where $m = w = 0$ at the hand ends. In the other cases of the limit conditions one associates with (12), (14) either (19), (25) a) for a free end or (16) a) and (24) for a recess end. Let's write (12) , (14) and (24) as follow:

$$m_i = \frac{1}{2}(m_{i-1} + m_{i+1}) + \frac{1}{2}\Delta m_i + \frac{h}{2}\Delta m_i^\xi + \frac{h^2}{24}\left[{}^d p_{i-1} + 10\,{}^g p_i + {}^g p_{i+1}\right] + \frac{5h^2}{24}\Delta p_i + \frac{h^3}{24}\Delta p_i^\xi \quad (26)$$

$$v_i = \frac{1}{2}(v_{i-1} + v_{i+1}) + \frac{h^2}{24}\left[\gamma_{i-1}\,{}^d m_{i-1} + 10\gamma_i\,{}^g m_i + \gamma_{i+1}\,{}^g m_{i+1}\right] + \frac{5h^2\gamma_i}{24}\Delta m_i + \frac{h^3\gamma_i}{24}\Delta m_i^\xi \quad (27)$$

For the rigidly fixed-hand end we find m from (21) as follows:

$$m_i = -\frac{\gamma_i + \gamma_{i+1}}{4\gamma_i + h\gamma_i^\xi} m_{i+1} - \frac{h^2\gamma_i}{4\gamma_i + h\gamma_i^\xi}(5\,{}^d p_i + {}^g p_{i+1} + hp_i^\xi - \frac{12}{4\gamma_i + h\gamma_i^\xi}(v_i - v_{i+1}) + \frac{1}{h}v_i^\xi \quad (28)$$

The resolution of the equations thus obtained makes it possible to determine m and w. From (8a) one can determine all the values of M and W.Thus digital resolution gives complete results. All parameters of the stress state of the beam can be determined.

4 NUMERICAL VALIDATION OF THE THEORY

4.1 Example Number 1

Consider a beam [3] shown on Figure 2 and Figure 3 (top view) of variable thickness $b(Z) = \left(1 + \frac{Z}{l}\right) b_0$. The beam is simply supported at its left-hand end and rigidly fixed at its right-hand end. The beam is under the action of uniformly distributed load q; b_0 is the beam width at section A, l is the span of the beam.

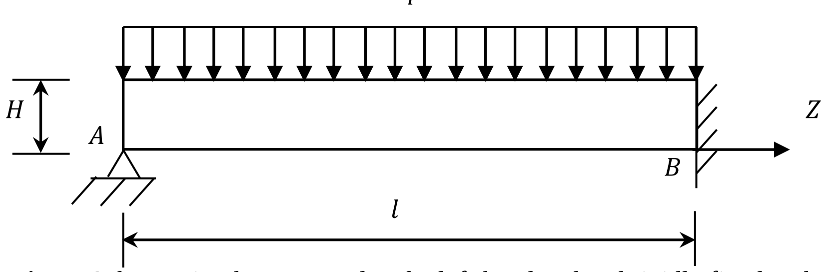

Figure 2: beam simply supported at the left-hand end and rigidly fixed at the other hand end under the action of a uniformly distributed load

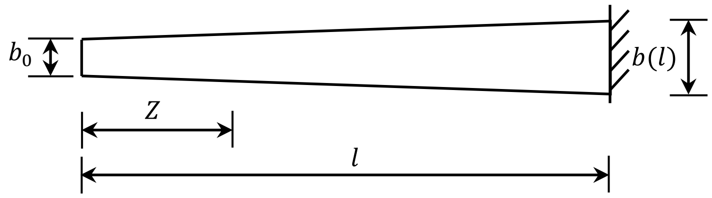

Figure 3: Top view of the beam of variable thickness

Considering that $\Delta m = \Delta m^\xi = \Delta p = \Delta p^\xi = 0$, $p = \frac{q}{q} = 1$ we can write (12), (14) as follow:

$$m_{i-1} - 2m_i + m_{i+1} = -h^2\,; \quad (29)$$

$$v_{i-1} - 2v_i + v_{i+1} = -\frac{h^2}{12}\left[\gamma_{i-1}m_{i-1} + 10\gamma_i m_i + \gamma_{i+1}m_{i+1}\right]; \quad (30)$$

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where $\gamma = \frac{EJ_0}{EJ} = \frac{b_0}{b} = \frac{b_0}{(1+\xi)b_0} = \frac{1}{1+\xi}; \quad \xi = \frac{Z}{l}.$ (31)

At the left-hand end A, the deflection and bending moment are equal to zero:

$v_i = 0, \; m_i = 0.$ (32)

According to the limit condition at the rigidly fixed right-hand end B the rotation angle is equal to zero. Thus we can write down (21) with replacement $i+1$ by $j-1$and at the same time $v_i = 0$, $p^{\xi} = 0$ while γ^{ξ}, v^{ξ} change the sign to the opposite:

$$-v_i^{\xi} = \frac{1}{h} v_{i-1} + \frac{h\gamma_i}{12}\left[(4 - \frac{h\gamma_i^{\xi}}{\gamma_i})m_i + (1 + \frac{\gamma_{i+1}}{\gamma_i})m_{i+1}\right] + \frac{h^3}{12}(5p_i + p_{i-1}), \quad (33)$$

where $\gamma^{\xi} = \frac{-1}{(1+\xi)^2}$. (34)

Thus, having written down (29), (30) for internal mesh points, (33) while $v_i^{\xi} = 0$ at nod B, taking into consideration (31) we get a system of algebraic equations, the solution of which allows one to determine dimensionless deflections and dimensionless bending moments at any nod of the beam.

In table1, the first column shows the results obtained in [3], in columns 3,4,5,6, the results obtained at the middle of the span and on the rigidly fixed hand end under different partitions using the above equations. The found value of the bending moment at rigidly fixed hand end differs from the exact value by 1.72% when $n = 8$ and when $n = 32$ this value exceeds the exact value by 0.07%.

Table 1: values of the bending moment and deflection coefficients at the middle and at the rigidly fixed right-hand end of the beam of variable thickness

	[3]	exact solution	Finite	difference	equations	of SAM	
n(ч.разб.)	4	/	4	8	16	24	32
$10^2 . v_{cp}$	/	/	0.2565	0.3217	0.3298	0.3306	0.3308
$10. M_{cp}$	/	/	0.4724	0.5529	0.5630	0.5641	0.5643
m_B	- 0.1140	- 0.1370	- 0.1555	- 0.1394	- 0.1374	- 0.1372	- 0.1371

Let's perform the calculation of the same beam using (14). To take into consideration the limit conditions of the rigidly fixed hand end B, this equation must be written by replacing $i+1$, $i+2$ respectively by $i-1$, $i-2$ and at the same time $v_i = 0$ while γ^{ξ}, v^{ξ} change their sign to the opposite. Solving equations of the type (29), (30) taking into consideration the limit conditions, we obtain the results given in Table 2.

Table 2: values of the bending moment and deflection coefficients at the middle and at the rigidly fixed right-hand end of the beam of variable thickness

	[3]	Exact solution	Finite difference equations of SAM		
n (number of elelments)	4	/	4	6	8
$10^2 . v_{cp}$	/	/	0.3307	0.3309	0.3309
$10. M_{cp}$	/	/	0.5651	0.5646	0.5645
m_B	- 0.1140	- 0.1370	- 0.13690	- 0.13707	- 0.13710

Table 2 illustrates the non-monotonic convergence of the numerical solution to the exact one. We denote that the results reported in this table show that when $n = 4$ the value of m_B differs from the exact one only by 0.07%, whereas in Table 2 this result is obtained when $n = 24$. Thus, it should be noted the possibility of using the (14) in the calculations.

4.2 Example Number 2

Consider the case the cantilever shown in Figure 4, when the left-hand end is free and the right-hand end is rigidly fixed.

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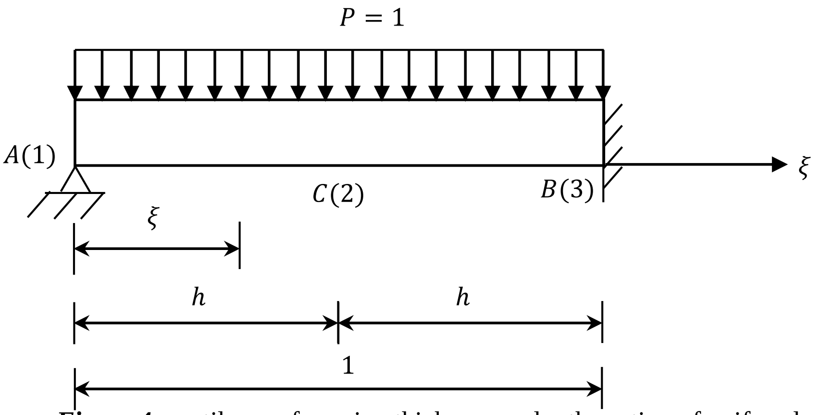

Figure 4: cantilever of varying thickness under the action of uniformly distributed load

Let's divide the beam Figure 4 into two equal sections, that is $h = \frac{1}{2}$. We denote the nods A, C, B respectively by 1, 2, 3. According to (31) $\gamma_1 = 1, \gamma_2 = \frac{2}{3}, \ \gamma_3 = \frac{1}{2}$ and according to (31) $\gamma_3^{\xi} = -1$

(35)

To calculate the bending moment at nod 2, it suffices to write (19) for nod 1 with respect to $p^{\xi} = 0$, $m_1 = 0$, $p_1 = p_2 = p = 1$, $h = \frac{1}{2}$, then $m^{\xi} = 0$: $2m_2 + \frac{1}{2}.\frac{1}{12}(5.1 + 1) = 0$ (36)

Next, we write the equation (29) for the nod 2 taking into consideration $m_1 = 0$, $h = \frac{1}{2}$:

$-2m_2 + m_3 = -0.25$ (37)

From (36) we find $m_2 = -0.125$; then m_2 substitute in (37) we get $m_3 = -0.5$. To calculate the deflections at the nods 1 and 2, it is necessary to write down (30) for the nod 2 and an equation of the type (14) for the nod 3 taking into consideration the above values of γ_1, γ_2, γ_3, γ_3^{ξ}. The obtained equations are solved together with the found value of m_2, m_3.Table 3 shows the results obtained using the difference equations of SAM and the exact solutions obtained by the method of direct integration.

Table 3: values of the bending moment and deflection coefficients at the left-hand end, at the middle and at the right-hand end of the cantilever of variable thickness

	n (nber of elts)	v_1	v_2	m_2	m_3
Finite difference equations of SAM	2	0.0573	0.0213	- 0.125	- 0.500
	4	0.0688	0.0232	- 0.125	- 0.500
Exact solution (anlytic method)		0.0700	0.0238	- 0.125	- 0.500

When the beam is divided into 4 equal parts, we get $v_1 = 0.0688$ at its free hand end, which is 1.74% less accurate, and in the middle of its span, the error is equal to 2.6%.

4.3 Example Number 3

Consider another example. The beam is simply supported at the two hand ends as shown in Figure 5. The calculation of such a beam is reduced to the solving systems of equations of the type (29), (30). Let's dividing the beam Figure 5 into two equal sections and write these equations for the nod 2, while taking into consideration (32), (35), $p = 1$, $h = \frac{1}{2}$: $-2m_2 = -\frac{1}{4}$;

$-2v_2 = -\frac{5}{36}m_2$ (38)

From (38) we find: $m_2 = 0.125$, $v_2 = 0.00868$. When $h = \frac{1}{10}$ $m_2 = 0.125$, $v_2 = 0.0088$.The exact solution (by the method of direct integration): $v_2 = 0.0088$.

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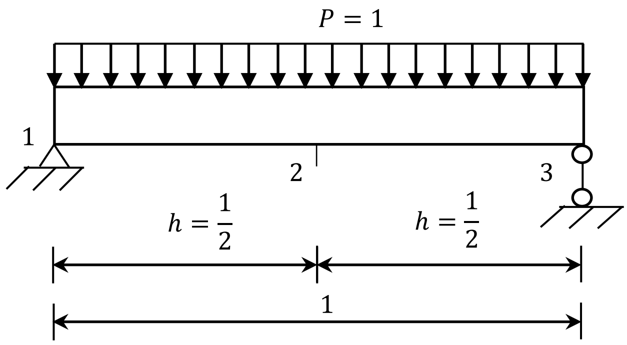

Figure 5: simply supported beam under the action of a uniformly distributed load

4.4 Example Number 4

Consider also the case when the same beam is rigidly fixed at both its hand ends and loaded in the middle of the span with a concentrated force Figure 6: $\Delta m^{\xi} = \frac{Q}{q_0 l} = 1$.

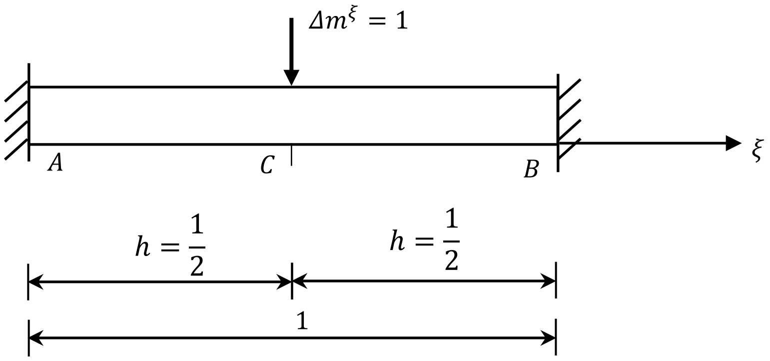

Figure 6: rigidly fixed beam of varying thickness under the action of a concentrated force

We divide the beam into 6, 8, 12, 16, 20, 24, 32 equal sections, that is $h = \frac{1}{6}$, $h = \frac{1}{8}$, $h = \frac{1}{12}$, $h = \frac{1}{16}$, $h = \frac{1}{20}$, $h = \frac{1}{24}$, $h = \frac{1}{32}$. Let's write (12), (30) for nods belonging to the internal mesh, (14) for nod A and the equation of this type for the nod B, while taking into consideration (31), (33), $p = 0$. Thus we obtain a system of equations, the solution of which allows us to determine m, v, at any nod of the beam. Table 4 shows the calculation results for the beam considered above, rigidly fixed at its two ends and loaded with concentrated force. In the second and third lines, the values of the dimensionless deflection and bending moment in the middle of the span are given for different beam splits. The fourth and fifth lines show the values of bending moments in the rigidly fixed ends for different values of h. *Table 5* shows the results of calculating the same beam using formulas like (21). As can be seen from the tables, the results are almost the same.

Table 4: values of the bending moment and deflection coefficients at the left-hand end, at the middle and at the right-hand end of the rigidly fixed beam of varying thickness

n (number of elements)	6	8	12	16	20	24	32
$10^3 . v_C$	4.909	4.705	4.159	3.903	3.774	3.701	3.628
$10. m_C$	0.827	1.038	1.162	1.202	1.220	1.230	1.238
$10. m_A$	-1.031	-1.098	-1.081	-1.065	-1.056	-1.050	-1.044
$10. m_B$	-1.095	-1.496	-1.556	-1.520	-1.501	-1.490	-1.478
$r_A + r_B$	0.3939	0.7689	0.9896	0.9980	0.9995	0.9999	0.9999

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Table 5: values of the bending moment and deflection coefficients at the left-hand end, at the middle and at the right-hand end of the rigidly fixed beam of varying thickness

n (number of elements)	6	8	12	16	20	24	32
$10^3 . v_C$	4.953	4.705	4.159	3.903	3.773	3.706	3.628
$10. m_C$	0.833	1.038	1.163	1.202	1.220	1.233	1.238
$10. m_A$	-1.040	-1.098	-1.082	-1.065	-1.055	-1.052	-1.044
$10. m_B$	-1.176	-1.496	-1.556	-1.520	-1.501	-1.494	-1.478
$r_A + r_B$	0.4547	0.8307	0.9914	0.9980	0.9995	0.9999	0.9999

According to the formulas of the type (19), (14), the dimensionless reactions at the supports r_A, r_B were calculated, when splitting the beam into 6, 8, 12, 16, 20, 24, 32 equal sections. The sums of the obtained results are given in the last line of Table 4, Table 5. It can be seen that we have good convergence when $n \geq 12$. According to Table 5: when $n = 12$ $r_A + r_B = 0.9914$ that is less than the transverse concentrated force in the mid-span of the beam ($\Delta m^\xi = 1$) only by 0.0086, and when $n \geq 20$ the sum of the dimensionless support reactions is almost equal to 1, that is, the transverse force $\Delta m^\xi = 1$ at the middle of the beam.

5. CONCLUSION

The algorithm is easy to implement and allows to solve problems of the calculation of bending beams of variable thickness with any kind of limit conditions. As it was said in [4], the algorithm can also take into consideration the finite discontinuities of all the parameters and their first derivatives. The verification of the integral equilibrium conditions shows that the results obtained are precise.

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