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Aerodynamic Admittance Function using Impulse Response Function Approach

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Abstract

An explanation described here is how the aerodynamic admittance function(AAF) is to derive by means of impulse response function(IMF) which is modified for two peaks (movement-induced and Karman Vortex-induced) intending to be able to deal with the response condition under higher reduced wind speed. By using the analogy of the Sears function and the flutter derivatives, the relationship between the aerodynamic derivatives(AD) and aerodynamic admittance function is clarified. The equivalent Sears function(ESF) is obtained through Fourier transform of IMF incorporate with some shape parameters. After AD has been approximated, the corresponding AAF, which is the square value of ESF, is achieved. This paper includes the verification that the AAF of thin airfoil estimated applying the formulation is found to agree well with thin airfoil Sears function proposed by Holmes and Liepmann.

Key Words: *Airfoil. Aerodynamic admittance function, Aerodynamic derivatives, Impulse response function, Sears function.*

1. INTRODUCTION

Understanding of aerodynamic response of a structure subjected by wind turbulence play an important role in serviceability and safety of those structures which are vulnerable to wind fluctuation. The Fifth of Tay Bridge, which was collapsed in 1879, was blown down by gust wind. The failure of Tacoma narrow bridge in 1940 is believed that the bridge was destroyed not by static wind loading but because of dynamic excitation.

To investigate the response of a structure to dynamic excitation, it is not enough to consider only the structural characteristics such as inertial, damping, stiffness and configurations, then how to handle the random vibration characteristics of time dependent aerodynamic force is necessary to perform aeroelastic analysis. For this purpose, Davenport (1962) introduced a spatial approach in frequency domain.

In buffeting analysis by this approach as shown in figure 1, wind speeds, pressure and resulting structural response are treated as stationary random process. Firstly, the spectrum of the aerodynamic forces is calculated from gust spectrum using a set of filters called aerodynamic admittance function. And then mechanical admittance function acts as a modification factor between the force spectrum and response spectrum, which in turn, the total mean square fluctuating response is computed.

In current wind engineering practice, strip and quasi-steady theories are generally employed in formulating analysis of wind effects on structures. The application of these assumptions permitted the representation of the wind pressure field on the building surface completely by the oncoming wind velocity field [1,2]. The aerodynamic admittance function(AAF), is actually a correction function for quasi-study assumption and the Fourier transformation from time domain into frequency domain.

The frequency domain approach for buffeting analysis has been widely used in the aeronautical field since the works of Küssner (1931) and Sears (1938). The latter obtained the aerodynamic admittance function relating the lift force and the vertical component of velocity for an infinitely thin streamlined section in invisible flow. In these conditions, the superposition principle of flow patterns holds and the thin airfoil theory can be applied to obtain in closed form the so-called Sears function.

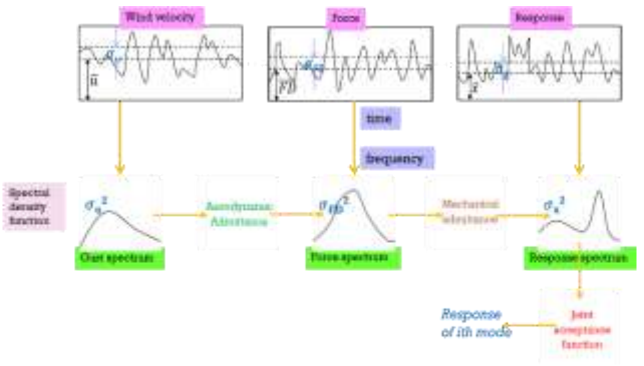

Figure. 1.1 Buffeting analysis procedure proposed by Dovenport

The same approach has been extended to Wind Engineering by Davenport (1962), who first introduced six aerodynamic admittance functions, relating drag, lift and moment to longitudinal and vertical turbulence components. Recent studies have introduced relationships among the aerodynamic admittance functions and the flutter derivatives (Scanlan and Jones 1999) [3].

It can be achieved evaluating aerodynamic admittance functions through the indicial approach (Von Karman and Sears 1938). The method has been introduced in literature for the assessment of the aerodynamic and aeroelastic forces acting on a thin airfoil, in nonuniform motion and potential flow. Generally, the indicial approach consists in the determination of the generalised forces (drag, lift and moment) arising on a body as a consequence of a step variation of the motion of the body (Wagner problem) or due to its transition through a step-variation of the flow field (Küssner problem). The aerodynamic admittance functions, introduced within a frequency domain representation of the force components, can also be estimated from the indicial functions through Fourier transform operations.

2. FORMULATION STEPS

An effective reduction of the cost and time extension of wind tunnel test and CFD to determine AAF is the analogy to the airfoil flutter theory with some derivatives; the procedure is described here. Lift force per unit span length due to sinusoidal vertical gust on is expressed using Sears function:

$$L_w(t) = \frac{1}{2}\rho BU\left(\frac{dC_L}{d\alpha}\right)_{airfoil}.S(k).w(t) \tag{1}$$

where, $L_w(t)$ = lift force per unit span length due to vertical gust wind (N/m)

ρ=air density (kg/m^3)

U=mean wind speed (m/s)

$\left(\frac{dC_L}{d\alpha}\right)_{airfoil}$ =slope of lift force coefficient of a flat plate or an airfoil

$w(t)$=vertical gust wind (m/s)

S(k)=Sears function

K=reduced frequency

Similarly, lift force per unit span length due to sinusoidal vertical gust on a 2-D bluff body can be expressed as follows, if there is an aerodynamic admittance function of the 2-D bluff body which is equivalent with the Sears function:

$$L_w(t) = \frac{1}{2}\rho BU\left(\frac{dC_L}{d\alpha}\right)_{bluffbody}\phi_{eq}(k)w(t) \tag{2}$$

where, ϕ_{eq}(k)=equivalent Sears function

On the other hand, self-excited force due to sinusoidal heaving motion on a 2-D bluff body is expressed using Scanlan's aerodynamic derivatives for heaving:

$$L_h(t) = \frac{1}{2}\rho BU^2\left(KH_1^*\frac{h'}{U} + KH_4^*\frac{h}{U}\right) \tag{3}$$

where, $L_h(t)$=lift force per unit span length due to heaving motion (N/m)

H1*, H4*= aerodynamic derivatives

h(t)=heaving displacement (m)

h'(t)=heaving velocity (m/s)

The above self-excited lift force can be described using an aerodynamic transfer function as:

$$L_h(t) = \frac{1}{2}\rho BU\left(\frac{dC_L}{d\alpha}\right)_{airfoil}.[\chi_{LhR}(K).h'(t) + \omega_h\chi_{LhI}(K).h(t)] \tag{4}$$

where, χ_{Lh}=aerodynamic transfer function

By comparing with equation (3) and (4), the following relationship is obtained:

$$KH_1^* = \left(\frac{dC_L}{d\alpha}\right)_{bluffbody} \chi_{LhR}(K)$$
$$KH_4^* = \left(\frac{dC_L}{d\alpha}\right)_{bluffbody} \chi_{LhI}(K) \tag{5}$$

The condition in which both functions are equivalent can be explained as follows. If sinusoidal vertical gust w(t) is approaching and passing through a 2-D bluff body, the chordwise distribution of relative angle of attack w/U will be also sinusoidal and there will be a certain time lag of w/U at the trailing edge to that at the leading edge. On the other hand, if the same 2-D bluff body is in harmonic heaving motion in uniform flow, the distribution of relative angle of attack $\dot{h}/U$ will be also uniform along chordwise direction.

The wavelength λ of the sinusoidal gust is expressed as

$$\lambda = \frac{2\pi U}{\omega_b} \tag{6}$$

and the condition that the distribution of the relative angle of attack w/U is almost uniform along chordwise direction can be described as λ >> B which yields the following condition, k << π or K << 2π. This condition means that if the equivalent aerodynamic admittance ϕ_{eq}(k) can be obtained by the aerodynamic derivatives H1*, H4* in relative higher reduced wind velocity region.

$$V_r = \frac{U}{nB} = \frac{1}{K} > \frac{1}{2\pi} \approx 0.16 \tag{7}$$

It is expected that this condition covers the across wind response of tall buildings due to wind turbulence.

The impulse response function is approximated by the following exponential

$$I_{eq}(\tau) = x_1 \exp(-x_2\tau) + x_3 \exp(-x_4\tau) \quad \text{for } \tau \geq 0 \text{ and } 0 \text{ for } \tau \leq 0 \tag{8}$$

By integrating equation (6) with respect to τ, the aerodynamic indicial function is obtained as:

$$\psi_{eq}(\tau) = -\frac{x_1}{x_2}\exp(-x_2\tau) - \frac{x_3}{x_4}\exp(-x_4\tau) + C \tag{9}$$

where, $x_1 > 0$, $x_2 > 0$, $x_4 > 0$.

According to the quasi-steady condition $\psi_{eq}(\alpha)$=1.0, the coefficients x_1, x_2, x_3, x_4 are mutually dependent and the following constraint condition has to be satisfied:

$$1 - \frac{x_1}{x_2} - \frac{x_3}{x_4} = 0 \tag{10}$$

The equivalent Sears function is obtained through Fourier Transform as:

$$\phi_{eq}(k) = \phi_{eqR}(k) - i\,\phi_{eqI}(k)$$
$$\phi_{eq}(k) = \left[\frac{x_1x_2}{{x_2}^2 + k^2} + \frac{x_3x_4}{{x_4}^2 + k^2}\right] - i\left[\frac{x_1k}{{x_2}^2 + k^2} + \frac{x_3k}{{x_4}^2 + k^2}\right] \tag{11}$$

Assuming $x_1/x_2=X_1$, $x_3/x_2=X_3$ and $k/x_2=\kappa$, the equation (12) becomes

$$\phi_{eq}(k) = \left[\frac{X_1}{1 + \kappa^2} + \frac{(1 - X_1){X_3}^2}{{X_3}^2 + \kappa^2}\right] - i\left[\frac{X_1\kappa}{1 + \kappa^2} + \frac{(1 - X_1)X_3\kappa}{{X_3}^2 + \kappa^2}\right] \tag{12}$$

The equivalent Sears function in absolute value is

$$|\phi_{eq}(k)| = \sqrt{[\phi_{eqR}(k)]^2 + [\phi_{eqI}(k)]^2}$$
$$|\phi_{eq}(k)| = \sqrt{\frac{(X_1X_3 - X_1 - X_3)^2\kappa^2 + {X_3}^2}{(1 + \kappa^2)({X_3}^2 + \kappa^2)}} \tag{13}$$

And equation (5) also becomes

$$\phi_{eqR}(k) = \frac{KH_1^*}{\left(\frac{dC_l}{d\alpha}\right)_{bluffbody}}$$
$$\phi_{eqI}(k) = \frac{KH_4^*}{\left(\frac{dC_l}{d\alpha}\right)_{bluffbody}} \tag{14}$$

According to equation (11) and (14), the following relationship is obtained

$$\left[\frac{x_1x_2}{{x_2}^2 + k^2} + \frac{x_3x_4}{{x_4}^2 + k^2}\right] = KH_1^*$$
$$\left[\frac{x_1k}{{x_2}^2 + k^2} + \frac{x_3k}{{x_4}^2 + k^2}\right] = KH_4^* \tag{15}$$

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Finally, the curve H1* can be approximated by determining shape parameters with trial and error manner, the equivalent Sears function will be determined using H1*(K) which is necessary to evaluate the cross-wind response.

3. RESULT AND DISCUSSION

By means of the value of X1 and X3 are approximated trial and error manner, figure (3.1) is obtained. There is three area which are divided according to the general characteristic of the curves of H1* and $\phi_{eq}(k)$.

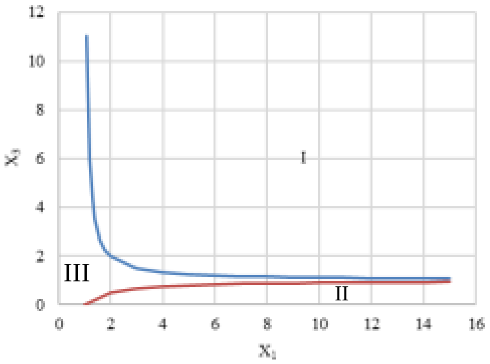

Figure. 3.1 Area classification according to curve shape

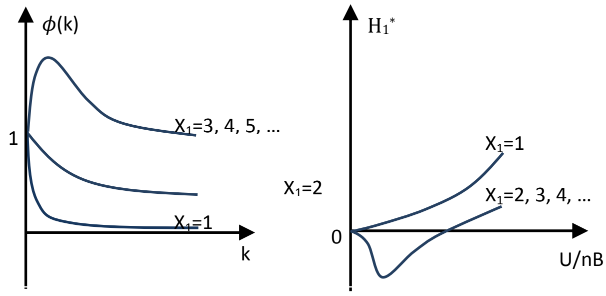

Figure. 3.2. The potential characteristics of the curve for various X_1

The figures (3.3) and (3.4) show that the curve of $\phi_{eq}(k)$ of X1=1 for all X3 is the same for each x2 value and also the same manner in the case of H1*. The curve of $\phi_{eq}(k)$ and H1* of X3=1 for all X1 is also the same for each x2 value in area (III). It is clear that plotting the value of functions $\phi_{eq}(k)$ using the X1 and X3 value in area (II) for each x2 shows greater than 1 and then decrease with increasing the reduced frequency. All the aerodynamic derivatives start from 0 and go up with positive values as shown in figure (3.5). Whereas, in area (I) the aerodynamic admittance function and the derivative shows in figure (3.6). Generally, the $\phi_{eq}(k)$ value is greater than 1 and then decreasing with the increasing value of reduced frequency. Although the derivatives value is reducing firstly, the value increases to positive value in higher reduced velocity region. Summarizing, the respective curve shape for various X1 value are also describe in figure (3.2).

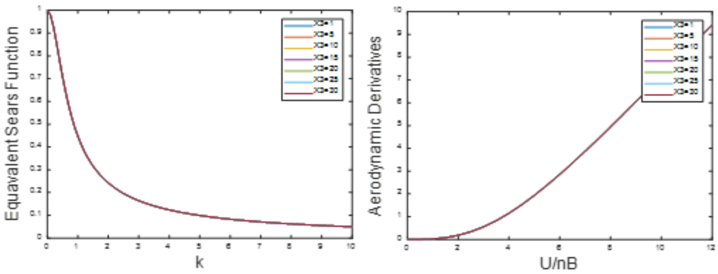

Figure. 3.3 Equivalent Sears function and aerodynamic derivative in area (III) X_1=1 for all X_3, x_2=0.5

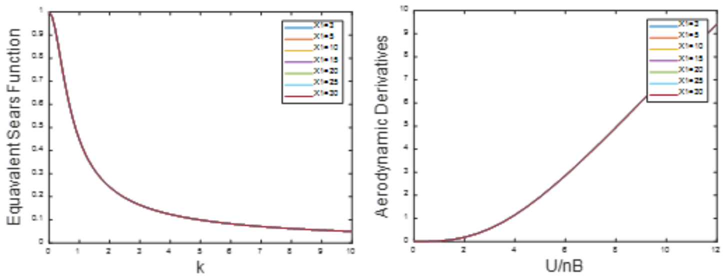

Figure. 3.4 Equivalent Sears function and aerodynamic derivatives in area (III) X_3=1 for all X_1, x_2=0.5

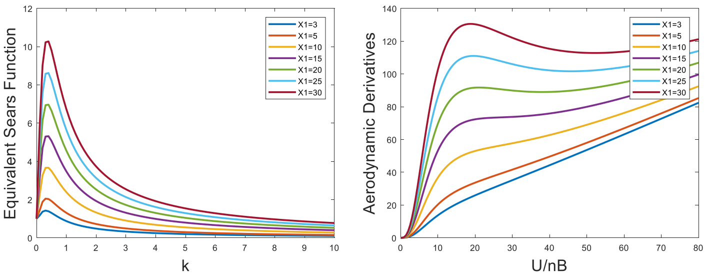

Figure.3.5 Equivalent Sears function and aerodynamic derivatives in area (II) X3=0.5 for all X1 except 1, x2=0.5

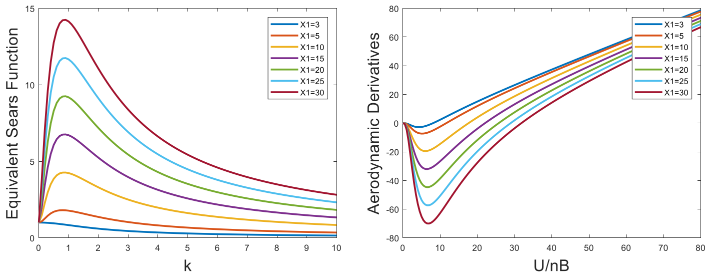

Figure.3.6. Equivalent Sears function and aerodynamic derivatives in area (I) X_3=3 for all X_1 except 1, x_2=0.5

3.1 Application to airfoil

Because the H1* value of thin airfoil is starting from 1 and decreased with increasing value of U/nB, the potential curve shape must be in the region of area (I). So, the most possible curve shape is obtained with the approximated value of x2, X1, and X3. The equivalent Sears function derived from the aerodynamic derivatives in the above trial and error manner is compared with that approximated by Liepmann and Holmes as shown in figure (3.7).

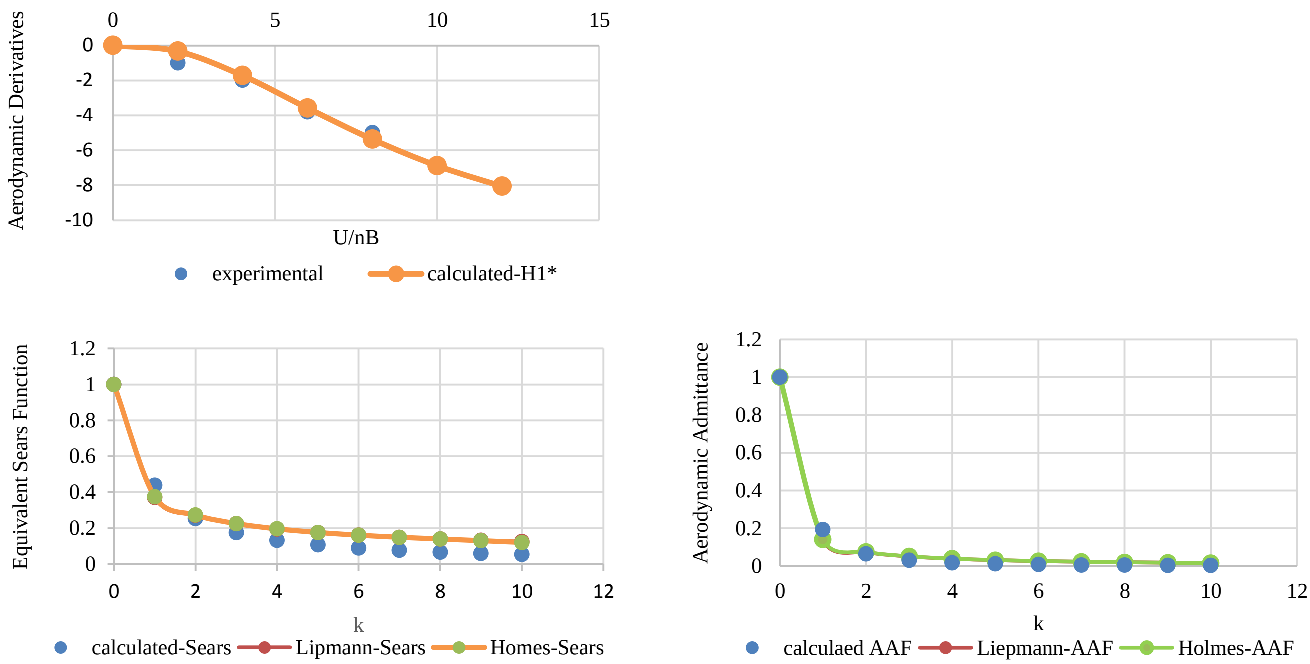

Figure. 3.7 Comparison on AD, ESF and AAF of airfoil

It can be easily seen that the equivalent Sears function from aerodynamic derivatives is a little bit different from those of Liepmann and Holmes in lower reduced frequency range of reduced frequency, but it is almost the same as in higher region of reduced frequency.

3. CONCLUSION

The main goal of the present paper is to describe a relationship between aerodynamic admittance function and flutter derivatives involving some parameters to identify the possible curve shapes.

It is proved that the equivalent Sears function shows the same characteristic shape for each x_3 under area (III). It is also true for the aerodynamic admittance H_1^*. Under area (II), all the ESF are greater than one at first and then decreased while all H_1^* are in positive value. In contrast, H_1^* in area (I) are negative in smaller reduced velocity and then gradually increased in accordance to greater V_r. For, ESF, the characteristics is the same as in cases under area (II). The reason of that all the ESF starts form 1 and H_1^* starts from zero under all positive values of X_1 and X_3 is assured that the above approach of approximating AAF is worth to use for other cross-sections.

For thin aifoil, the equivalent Sears function and aerodynamic admittance function can be said well consent to thin airfoil theory suggested by Liepmann and Holmes.

Moreover, result of provision of two terms in the impulse response function utilized in this paper, the mathematical model available here can accommodate the conditions under higher reduced wind velocity region where the response of the structure has two peaks (motion-induced and Karman vortex).

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