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Developing a Stochastic Model for Studying and Simulating Sediment Transport in Ports and Harbors

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ABSTRACT

A particle model to describe and predict sediment transport in shallow water is developed with the use of random walk models. The model is developed by showing consistency between the Fokker-Plank equation and the Advection diffusion equations. Erosion and deposition process in the model are developed probabilistically where the erosion term is considered to be a constant and deposition term is taken as a function by relating sediment settling velocity and diffusion coefficient. Eventually, we simulated the particle model by considering three environment tests. In each environment test the simulations show the distribution of particle and the position of each particle at any given time t. The simulations also show the particles that will finally remain in suspension state and the particles that will be deposited during the transport process following the deployment of 10,000 particles. It was also established that there is uniform distribution of particles in test environment I and III and a linear dependence between the number of particles in different grid cell and the water depth in test environment II.

Key Words: *Stochastic differential equation, Random walk model, Fokker-Plank equation, Advection diffusion equation, Brownian motion.*

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1. INTRODUCTION

Sediment transport deals with movement of fragmented materials (particles) by flowing water. There are three types of sediment transport which are bedload, saltation and suspension. Bedload transport occurs when sediment grain travel along the bed. Saltation occurs when a single grain whose length which is proportional to its diameter jump over the bed. Sediment is transported in suspension form when the flux is enough for sediment particle to reach a considerable height over the bed. These variations in movement are due to number of factors. To start with, variation of particle properties cause variation in particle movement. Such properties can be in terms of size, shape, mass or contents. Particle movement can also be affected by force exerted on it. For example, particle deposition and erosion depend on the force exerted on it, low force results to insufficient velocity of the particle and hence deposition. When high force is exerted velocity becomes large enough to cause erosion. More important sediment transport is turbulent in nature and hence this randomness results to variability in particles movement. In the presence of extreme flows, sediment transport has a significant negative impact, can cause environmental damages, poor water quality, destruction of land resources and structures, reduction in water depth in ports and harbours and ultimately economic instability. With these accounts there is a need of clear understanding of sediment transport to control effects associated with sediment transport. The focus of this study is to develop a stochastic model for sediment transport that can be used to predict the transport rate of sediments and change in water depth in ports and harbours since accumulation of sediments in harbours and ports continues to be a challenge in ports planning and operations [1]. Among other things accumulation of sediments in ports and harbours leads to reduced water depth which hinders large ships to dock. Thus, before planning and carry out dredging activities there is a need for understanding sedimentation rates. High rates of sedimentation bring the necessity of frequent dredging, this bring high-cost implication to ports and harbours operations. In the following section we introduce a mathematical model that deals with sediment transport and shallow water equations. In the following section we introduce a mathematical model that deals with sediment transport and shallow water equations.

2. SHALLOW WATER FLOW EQUATIONS

To describe transport problems in ports and harbors, the particle model requires input such as water depth $H(x, y, t)$, water level ξ , water flow velocities $[U(x, y, t),\ V(x, y, t)]^T$ and so forth, with the assumption that these inputs satisfy shallow water flow equations. The mass and momentum exchange of sediment mixture are given by the following shallow water equations (1-3).

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + g\frac{\partial \xi}{\partial x} - fV + g\frac{U(U^2+V^2)^{\frac{1}{2}}}{(C_z)^2 H} = 0 \qquad (1)$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + g\frac{\partial \xi}{\partial y} + fU + g\frac{V(U^2+V^2)^{\frac{1}{2}}}{(C_z)^2 H} = 0 \qquad (2)$$

The rise and fall of the free surface are given by the continuity equation since over the vertical the velocity is considered to be uniform.

$$\frac{\partial H}{\partial t} + \frac{\partial (UH)}{\partial x} + \frac{\partial (VH)}{\partial y} = 0 \qquad (3)$$

Where,

t, x and y are time and cartesian coordinate in two dimensional, U and V are the velocity in $x-$ and $y-$direction respectively, $H = h + \xi$ is the total depth, ξ is the water level with respect to a reference, h represent depth of the water with respect to a refence, C_z is the bottom friction coefficient known as Chezy coefficient, g represent gravity of acceleration and f is the Coriolis parameter.

3. EULERIAN SEDIMENT TRANSPORT MODEL

The Eulerian transport model is used in this formulation to describe the dynamics of suspended particles. The model developed in this section by considering non-cohesive type of sediment particles. This is similar to that in [2], however with modification on the deposition parameter γ. In [2], γ is considered as a constant with the value approximated to $4 \times 10^{-3} s^{-1}$ for fine sand. In this work, γ is considered to be a function that relates γ to settling velocity (ω_s) and a diffusion coefficient (K) as done in [3]. The Eulerian model is

$$\frac{\partial (HC)}{\partial t} + \frac{\partial (HUC)}{\partial x} + \frac{\partial (HVC)}{\partial y} - \frac{\partial}{\partial x}\left(K\frac{\partial HC}{\partial x}\right) - \frac{\partial}{\partial y}\left(K\frac{\partial HC}{\partial y}\right) = -\gamma HC + \varepsilon(U, V).\lambda_s \qquad (4)$$

Where,

γ is the deposition coefficient, $\varepsilon(U, V) = (U^2 + V^2)(m^2 s^{-2})$ is a function of flow velocities, λ_s is the erosion coefficient, $\varepsilon(U, V).\lambda_s$ models erosion of sediment particles also $\varepsilon(U, V).\lambda_s$ is the parameterization of the particle pick up function. The term γHC models the deposition of sediment. $\lambda_s = 0.0001(kgm^{-4}s)$ as reported by [4] and γ is given by equation (5) below

$$\gamma = \frac{{\omega_s}^2}{K} \qquad (5)$$

Where,

ω_s is the settling velocity of naturally sediment which is given by equation (6) below

$$\omega_s = \sqrt{(13.95\frac{v}{d})^2 + 1.09(\frac{\rho_s}{\rho_w} - 1)gd} - 13.95\frac{v}{d} \qquad (6)$$

The settling velocity is estimated in m/s , v represents water viscosity, d is the sediment diameter, ρ_s represent sediment densities, ρ_w represents densities water and g represents the gravity acceleration. For dimensionless particles the diameter of the sediment is given by equation (7) below.

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$$d_* = d_{50}\left[\left(\frac{\rho_s}{\rho_w} - 1\right)\frac{g}{v^2}\right]^{\frac{1}{3}} \quad (7)$$

Where,

d_{50} represents the median of diameters. The diffusion coefficient K in equation (5) is taken as $0.01m^2s^{-1}$ following the study of [5] who performed tracer experiment to estimate diffusion values.

4. A PARTICLE MODEL FOR SEDIMENT TRANSPORT IN PORTS AND HARBOURS

A particle model is an effective way of describing and predicting sediment transport with the use of random walk models [6] and [7] . Random walk model is a stochastic differential equation which is used to describe path-valued process which have two parts, the drift part and a stochastic (diffusive) part.

4.1 Integration of particle movement

In this section, the following 2-dimensional stochastic differential equation is developed

$$dX(t) \overset{It\hat{o}}{=} \left[U + \frac{K}{H}\left(\frac{\partial H}{\partial x}\right) + \frac{\partial K}{\partial x}\right]dt \quad + \sqrt{2K}dB_1(t) \quad (8)$$

$$dY(t) \overset{It\hat{o}}{=} \left[V + \frac{K}{H}\left(\frac{\partial H}{\partial y}\right) + \frac{\partial K}{\partial y}\right]dt \quad + \sqrt{2K}dB_2(t) \quad (9)$$

Where, $B_1(t)$ and $B_2(t)$ are Gaussian Brownian process, $K(x,y,t)$ stands for horizontal dispersion coefficient of sediment. According to [8] horizontal dispersion coefficient is approximated to i.e. $K = \mathcal{O}(10-100)m^2/s$. $U(x,y)$ and $V(x,y)$ represents flow velocities along the x and y direction respectively which are given in m/s, $H(x,y)$ stands for averaged water depth plus relative water levels due to waves, $dB_1(t)$ and $dB_2(t)$ represents independent increment of Brownian motion with mean $(0,0)^T$ and covariance $\mathbb{E}[dB_1(t)dB_2(t)^T] = Idt$ $where$ I stands for identity matrix.

4.1 Deposition of sediment particles

In this section, binary state is used to describe the state of sediment particle at any specified time t.

$$S_t = \begin{cases} 1 & \text{suspension state.} \\ 0 & \text{deposition state.} \end{cases}$$

If the particle is in suspension state, our interest is on the transition from state 1 to state 0. The following equation can be used to model this transition in continuous form;

$$\frac{dP(S_t=1)}{dt} = -\gamma.P(S_t = 1), \quad \text{initially } P(S_0 = 1) = 1 \quad (10)$$

Where, γ represent deposition coefficient given by equation (4.5) and $P(S_t = 1)$ is the probability that, the state of the particle at time t is 1. Evolution of the particle in flow is given by the transition probability equation (11) below.

$$P(S_{t+\Delta t} = 1|S_t = 1) = P(S_0 = 1).[1 - \gamma.\Delta t] \quad (11)$$

$$= [1 - \gamma.\Delta t]$$

When the flow fields and turbulence patterns are assumed to be constant during the time step period, the probability that a particle will be sedimented is given by the following equation (12) below;

$$P(S_{t+\Delta t} = 0|S_t = 1) = 1 - P(S_{t+\Delta t} = 1|S_t = 1) \quad (12)$$

$$= \gamma.\Delta t$$

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4.2 Suspension of sediment particles

Group of particles concentration at a particular location are always represented by mass. Since the source term is included in the model, the expected number of suspended particles in a grid cell i, j the expected number of particles at a given time t, will be given by equation (13) through drawing a number from a Poisson distribution function.

$$enp(i,j,t) = \frac{\Delta x . \Delta y \Delta . \Delta t . (U^2 + V^2) . \lambda_s}{\mathcal{M}_p} \qquad (13)$$

Where,

Δx = width of the grid cell along x − direction

Δy = width of the grid cell along y − direction

Δt = Is the time step size

λ_s = Is the erosion coefficeient.

$\mathcal{M}_p$ = Is the mass of each particle.

5. The connection between the Fokker-Plank equation and Eulerian transport model

To show the relationship between the particle model (8-9) and the Eulerian transport model we have to consider the following. First, we have to assume that the mass expectation of a particle at position (x, y) at a time t is given by equation (14) which is known as mass density of a particle per unit area.

$$\langle m(x,y,t) \rangle = p(x,y,t) . P(S_t = 1) \qquad (14)$$

To derive the Fokker-Plank equation that includes sedimentation and suspension state of a particle, we have to consider the following. Let K be the diffusion coefficient as discussed in section (3), $\langle m(x,y,t) \rangle$ for mass density of a particle per unit area, $p(x, y, t)$ for probability density function of the particle position and $P(S_t = 1)$ for probability of the particle in suspension state. By using SDEs (8-9) the probability $p(x, y, t)$ results due to the Fokker Plank equation [9].

$$\frac{\partial p(x,y,t)}{\partial t} = \frac{-\partial}{\partial x}\left[\left(U + \frac{K}{H}\frac{\partial H}{\partial x} + \frac{\partial K}{\partial x}\right) . p(x,y,t)\right] - \frac{\partial}{\partial y}\left[\left(V + \frac{K}{H}\frac{\partial H}{\partial y} + \frac{\partial K}{\partial y}\right) . p(x,y,t)\right]$$

$$+ \frac{1}{2}\frac{\partial^2}{\partial x^2}(p(x,y,t) . 2K) + \frac{1}{2}\frac{\partial^2}{\partial y^2}(p(x,y,t) . 2K) \qquad (15)$$

In this section, we extended the model (8-9) by including erosion and deposition terms. Starting with differentiating equation (15) with respect to time t, the Fokker-Plank equation can be derived as follows;

$$\frac{\partial}{\partial t}\langle m(x,y,t) \rangle = P(S_t = 1)\frac{\partial}{\partial t}p(x,y,t) + p(x,y,t)\frac{\partial}{\partial t}P(S_t = 1) \qquad (16)$$

By using equation (12) in (16) we get

$$\frac{\partial}{\partial t}\langle m(x,y,t) \rangle = P(S_t = 1)\frac{\partial}{\partial t}p(x,y,t) - \gamma\, p(x,y,t) . P(S_t = 1) \qquad (17)$$

Adding erosion term to equation (17) we get,

$$\frac{\partial}{\partial t}\langle m(x,y,t) \rangle = P(S_t = 1)\frac{\partial}{\partial t}p(x,y,t) - \gamma\, p(x,y,t) . P(S_t = 1) + \varepsilon(U,V) . \lambda_s \qquad (18)$$

Next, we need to multiply $P(S_t = 1)$ on both sides of equation (15) we get (19)

$$\frac{\partial p(x,y,t)}{\partial t}P(S_t = 1) = \frac{-\partial}{\partial x}\left[\left(U + \frac{K}{H}\frac{\partial H}{\partial x} + \frac{\partial K}{\partial x}\right) . p(x,y,t)P(S_t = 1)\right]$$

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$$-\frac{\partial}{\partial y}\left[\left(V+\frac{K}{H}\frac{\partial H}{\partial y}+\frac{\partial K}{\partial y}\right).p(x,y,t)P(S_t=1)\right]+\frac{1}{2}\frac{\partial^2}{\partial x^2}\big(p(x,y,t).2KP(S_t=1)\big)$$

$$+\frac{1}{2}\frac{\partial^2}{\partial y^2}\big(p(x,y,t).2K(S_t=1)\big) \qquad (19)$$

Substituting equation (19) into equation (18), we obtain the following Fokker-Plank equation (20) with erosion and deposition term.

$$\frac{\partial}{\partial t}\langle m(x,y,t)\rangle=\frac{-\partial}{\partial x}\left[\left(U+\frac{K}{H}\frac{\partial H}{\partial x}+\frac{\partial K}{\partial x}\right).\langle m(x,y,t)\rangle\right]$$

$$-\frac{\partial}{\partial y}\left[\left(V+\frac{K}{H}\frac{\partial H}{\partial y}+\frac{\partial K}{\partial y}\right).\langle m(x,y,t)\rangle\right]+\frac{1}{2}\frac{\partial^2}{\partial x^2}(2K.\langle m(x,y,t)\rangle)$$

$$+\frac{1}{2}\frac{\partial^2}{\partial y^2}(2K.\langle m(x,y,t)\rangle)-\gamma\langle m(x,y,t)\rangle+\varepsilon(U,V).\lambda_s \qquad (20)$$

The particle concentration $C(x,y,t)$ given in kg/m^3 is related to this mass of a particle at position (x,y) through equation (21) given below.

$$C(x,y,t)=\frac{\langle m(x,y,t)\rangle}{H(x,y,t)} \qquad (21)$$

By substituting equation (21) into the Fokker-Plank equation (20) the Eulerian sediment transport equation (4) can be derived, equation (22)

$$\frac{\partial(HC)}{\partial t}+\frac{\partial(HUC)}{\partial x}+\frac{\partial(HVC)}{\partial y}-\frac{\partial}{\partial x}\left(D\frac{\partial HC}{\partial x}\right)-\frac{\partial}{\partial y}\left(D\frac{\partial HC}{\partial y}\right)=-\gamma HC+\varepsilon(U,V).\lambda_s \qquad (22)$$

Thus, we have shown that the model (8-13) is consistent with a well-known Eulerian sediment transport model (4). This means that, we can either solve equation (4) numerically or simulate the stochastic equation (8-9) for different many particles.

6. NUMERICAL APPROXIMATION OF THE PARTICLE MODEL

6.1 Deposition of sediment particles

In many cases numerical integration of particle position becomes a problem because of boundary conditions. Finding new location from the given location one may find the new location is outside the boundary. Thus, depending on the type of boundary conditions considered one may cause the phenomena to be physically impossible. In this work two types of boundary condition are considered which are open and closed boundary conditions. Closed boundaries, stands for boundaries related to the domains such as coast lines, banks and sea bed. Open boundaries depend on modeler's decisions to limit outside regions (artificially) which are of no interest or because at these locations there is no domain information. It is natural for particles to cross the open boundary and in this work such case is not within the scope of the model. The rules (i) and (ii) below apply when particles cross closed boundary for both the drift step and diffusive step of integration respectively.

i. Reduce the integration time to $2^{-n}\Delta t$ by halving the time step taken n −times, this makes remaining integration time to be $(1-2^{-n})\Delta t$. Thus, to complete the full time step Δt we need at least 2^n-1 steps. Note that this reduction process, affects only the current time step and the remaining sub steps remains unaffected. This approach is very useful in modelling shear along the coastline.
ii. Maintain step (i) above and restore the white noise process to its state in advance to invalidated integration step. Repeat halving process until the full time Δt time step is integrated without crossing the boundary.

6.2 Particle flux at open boundaries

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Particle flux depicts the difference of particles flowing into and outside the domain. As it is natural for particles to flow outside the domain, there is no need of controlling particles flowing out. Once the particle crosses an open boundary integration process stops as there is no domain information which is given. Following the mentioned reasons in this section we develop the model for the particles flowing in.

$$\mathrm{enp}(i,j,t) = \begin{cases} \dfrac{\Delta y.\Delta t.V.(U^2+V^2).\lambda_s}{\gamma \mathcal{M}_p} & \text{inflow parallel to y} - \text{axis} \\ \dfrac{\Delta x.\Delta t.U.(U^2+V^2).\lambda_s}{\gamma \mathcal{M}_p} & \text{inflow parallel to x} - \text{axis} \end{cases} \quad (23)$$

In each iteration, the above developed expectation value is used to find the possible number of sediment particles added in the domain boundary through drawing a number from Poisson distribution.

7. NUMERICAL EXPERIMENT

In this section we carried out simple experiments to simulate the particle model developed in section (3) to show the distribution of particles in suspension and deposition with the use of MATLAB. As many SDEs have no analytic solution, there are so many ways (schemes) which can be used to find numerical solution to systems of SDEs, these schemes include; Euler scheme, Milstein scheme and Heun Scheme. In this work the Euler scheme which has a strong order of convergence $\frac{1}{2}$ and weak order of convergence 1 was used to approximate the solution of the Itô SDEs. The discretization process of the two-dimensional SDEs was done in a similar approach as in [9]. The following is the Euler scheme used to approximate the solution of the SDEs for $t_0 < t_1 < t_2 \ldots t_N = T \ of \ [t_0, T]$.

$$\hat{X}(t_{n+1}) = \hat{x}(t_n) + \left[U + \left(\frac{K}{H}\frac{\partial H}{\partial x}\right) + \frac{\partial K}{\partial x}\right]\Delta t_n + \sqrt{2K}\Delta B_1(t_n) \quad (24)$$

$$\hat{Y}(t_{n+1}) = \hat{y}(t_n) + \left[V + \left(\frac{K}{H}\frac{\partial H}{\partial y}\right) + \frac{\partial K}{\partial y}\right]\Delta t_n + \sqrt{2K}\Delta B_2(t_n) \quad (25)$$

$$P_{n+1}(S_t = 1) = (1 - \gamma(x,y,t)\Delta_n)P_n(S_{t=1}) \quad (26)$$

where;

$\hat{X}(t_{n+1})$and $\hat{Y}(t_{n+1})$approximate X(t) and Y(t) respectively and $\hat{X}(t_0) = X(t_0) = x_0$ and $\hat{Y}(t_0) = Y(t_0) = y_0$ are the initial iterations of particles.

A domain of 20 by 20 was used to simulate the model x$\in [-10,10]$ $and\ y \in [-10,10]$. The simulation started by releasing 10,000 particles at the Centre (x, y) = (0,0). The time step used for all simulations of the model was 0.01 unless otherwise stated, values of the other variables used in the model are found in table 1. The boundary conditions were treated as described in section (6). To make effective simulation of the model the following three test environment which are similar to test environment suggested by [9] were considered. Various snapshots for particles positions in suspension and deposition for each test environment were taken at different times as shown in the following figures.

7.1 Test Environment I. U=3, V=2.5, H=10 and D is a 2D Gaussian Curve

In this first environment test, the water velocities and water depth are set to be constants and the dispersion coefficient D which varies with space. Below is the specific equation for Dispersion coefficient.

$$D(x,y) = 10 + 10\exp\left(0.03(x^2+y^2)\right) \quad (27)$$

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Figure 1 is the plot of equation (27) which shows that after a long time of simulation, particles in suspension and deposition will be uniformly distributed over the domain. The case is verified by figure 5 which is a simulation of the particle model with deposition and erosion term at large time t=10.

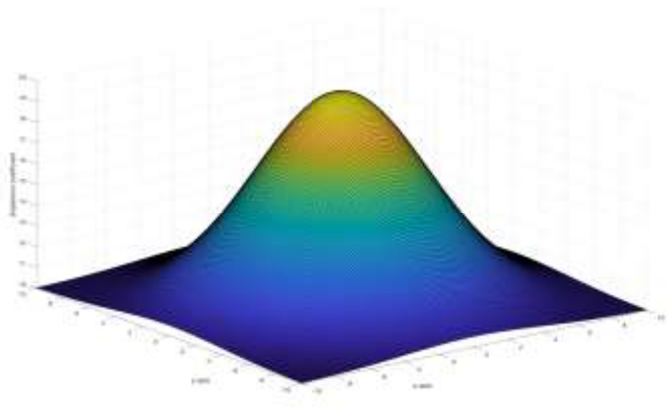

Figure 1. The Gaussian plot for dispersion coefficient.

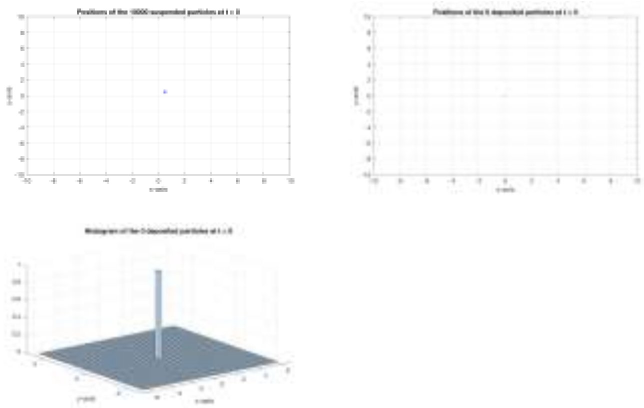

Figure 2. simulation of the particle model at t=0.

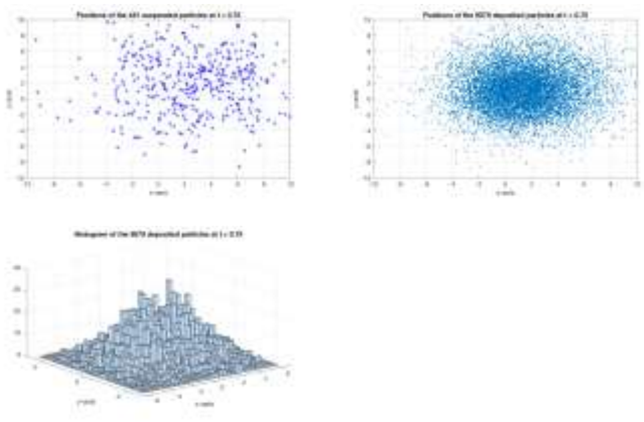

Figure 3. simulation of the particle model at t=0.5

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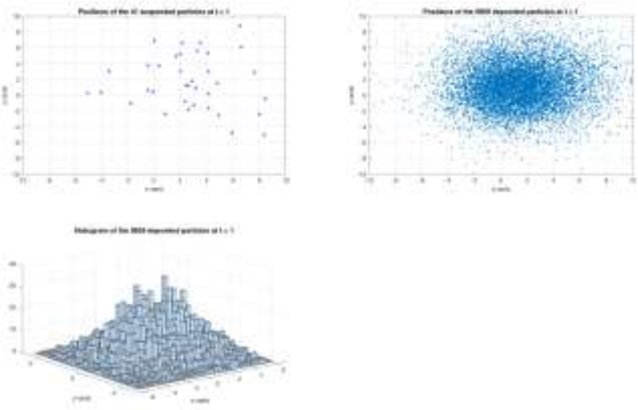

Figure 4. simulation of the particle model at t=1.

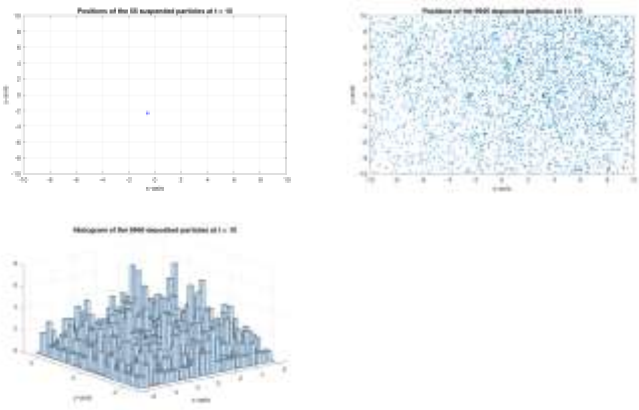

Figure 5. simulation of the particle model at t=10.

From figure 2 we can see that simulation started with the deployment of 10,000 particle at the middle of the domain (x, y) = (0,0), during this time (t=0) all 10,000 particles are assumed to be in suspension. Due to change in time the particles are distributed and some are deposited into different locations in the domain due to probabilistic condition developed in section (3). Each simulation indicates the number of particles deposited and remained in suspension for each iteration but also it shows the histogram of deposited particles to make the event clearer. To confirm that the simulation will give uniform distribution of particles in the domain for a large value of t, simulation of the particle model when t=10 was done and figure 5 was obtained, in this simulation the initial time was t=0 and the final time was t=10 and the time steps used was 10. Thus figure 5 proves that the particle distribution is uniformly distributed when simulation is done for a large value of t as it was expected from figure 1.

7.2 **Test Environment II. U=3, V=2.5, D=10 and H is a space varying depth**

In this test environment water flow velocities and dispersion coefficients are set to be constant while H is the space varying depth. The equation(28) below is the specific equation for depth.

$$H(x,y) = 10 - 5\tanh(x-4) + 5\tanh(x+4) \qquad (28)$$

Figure (6) is the plot of equation (28) which shows that after a long time of simulation, particles distribution is linearly dependent with depth. Thus, many particles will be more concentrated at the Centre of the domain compared to other parts of the domain. This case is verified by figure (7-10) which are simulations of the particle model with deposition and erosion term over various times.

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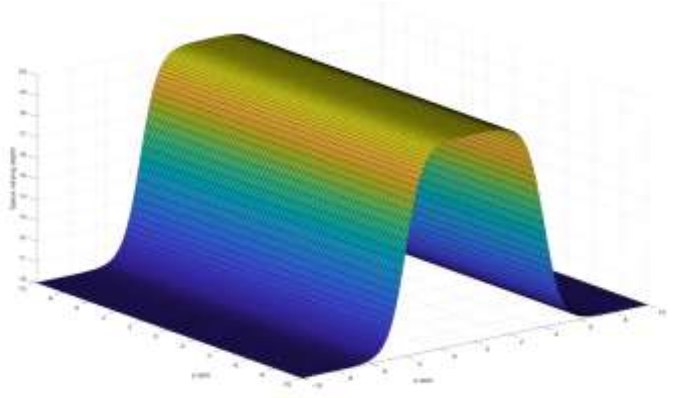

Figure 6. The plot of space varying depth.

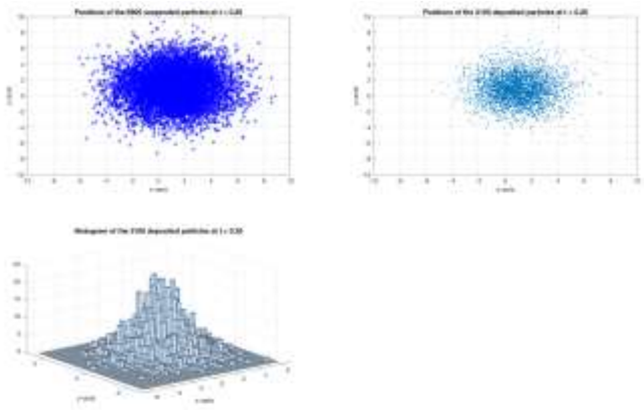

Figure 7. simulation of the particle model at t=0.25

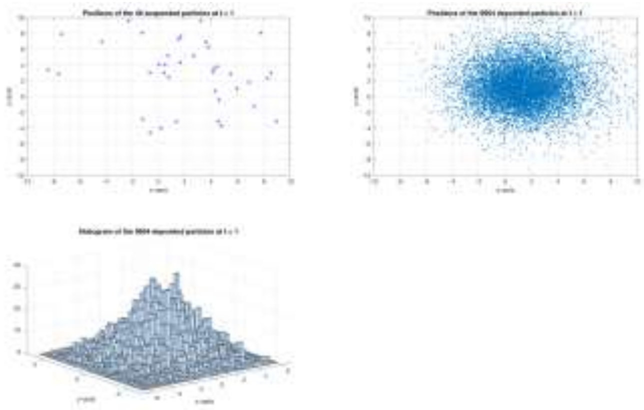

Figure 8. simulation of the particle model at t=1

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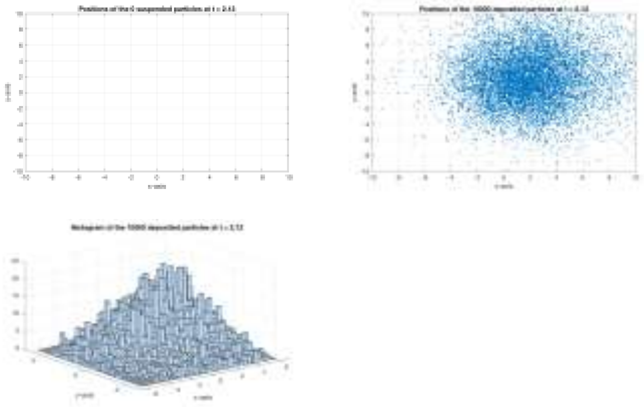

Figure 9. simulation of the particle model at t=2.13

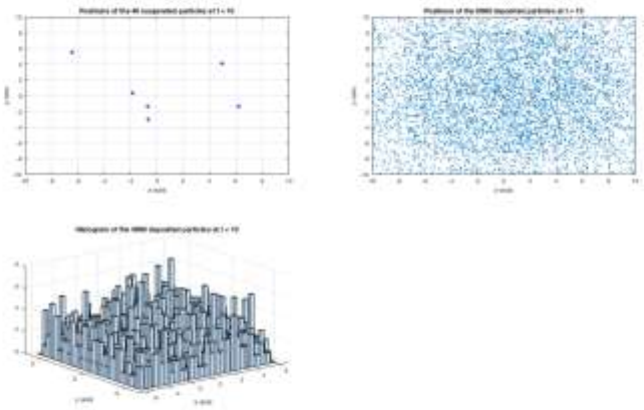

Figure 10. simulation of the particle model at t=10

Similarly, as in environment test **I**, we started simulation at t=0 with deployment of 10,000, as time goes on particles are distributed to different parts of the domain. As we can observe from results, we obtained through figure (7-10) particles are distributed in various parts of the domain but many particles are concentrated to the Centre of the domain due to deeper depth. Thus, we can conclude that the large the depth the more the particles will be deposited which is the expected result from figure 6.

7.3 **Test Environment III. D=10 and H=10 U and V are water flow velocities.**

In the third test environment we considered constant depth and Dispersion coefficient where water flow velocities are given by the following specific functions equation (29-30).

$$U(x,y) = \cos\left(\frac{\pi x}{20}\right)\sin\left(\frac{\pi y}{20}\right) \tag{29}$$

$$V(x,y) = -\sin\left(\frac{\pi x}{20}\right)\cos\left(\frac{\pi y}{20}\right) \tag{30}$$

In this last test environment, water flow is assumed to be in clockwise rotation so that velocities perpendicular to boundaries are zero, otherwise accumulation of particles will be near the boundary compared to other parts of the domain. Figure (11) shows the vector fields of water flow velocities in x and y directions.

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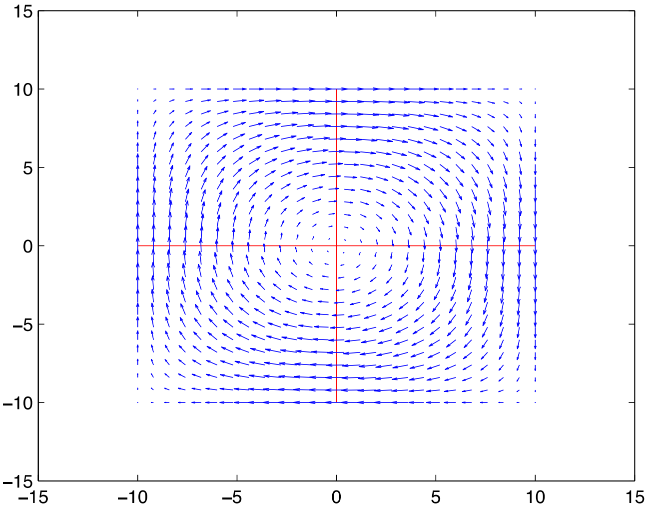

Figure 11. Vector fields of water flow velocities in x and y directions.

For a large time, simulation, particles in this test environment are expected to be uniformly distributed over the domain, this case is verified by figure (12) and figure (13).

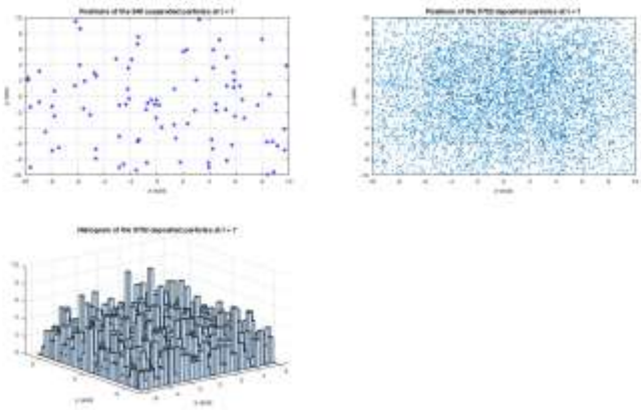

Figure 12. simulation of the particle model at t=7

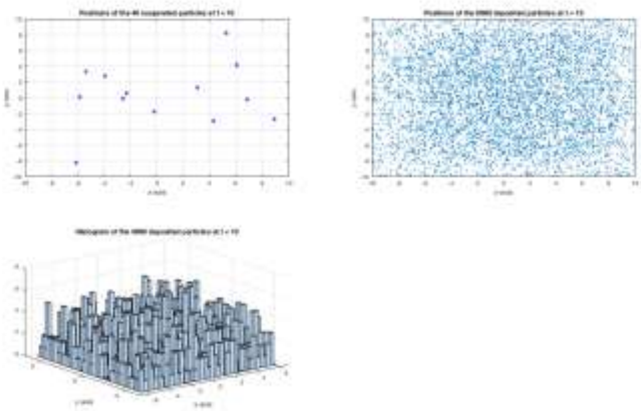

Figure 13. simulation of the particle model at t=10

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To confirm that the simulation will give uniform distribution of particles in the domain for a large value of t as expected, simulation of the particle model when t=7 and t=10 was done and figure 12 and figure 13 were obtained, in this simulation the initial time was t=0 and the final time was t=10 and the time steps used was 10. Thus, as expected figure 12 and 13 proves that particles are uniformly distributed when simulation is done for a large value of t.

Table 1. parameter used by the particle model to simulate sediment particle distribution.

Constants	Unit	Value
Water viscosity (ν)	-	$1 \times 2 \times 10^{-6}$
Water density (ρ_w)	Kg/m^3	1000
Gravity (g)	m/s^2	9.81
Particle density (ρ_s)	Kg/m^3	2650
Sediment diameter (d)	m	0.008
Flow velocity along x-direction (U)	m/s	3
Flow velocity along y-direction (V)	m/s	2.5
Dispersion coefficient (K)	m^2/s	0.01

8. CONCLUSION

In this research work the particle model were developed in which the deposition coefficient was considered as a function by relating the sediment settling velocity and dispersion coefficient. Numerical experiment for simulation of sediment particles in suspension and deposition have been discussed by considering various test environment. Finally, we simulated sediment particle distribution and various discussion on each test environment have been made. In the future, we plan to have a carefully study on Dar -es-salaam Ports and harbours and perform simulation by considering a realistic domain and other conditions so as to observe more interesting effects.

ABBREVIATIONS AND ACRONYMS

SDEs Stochastic Differential Equations

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