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Robust Statistical Normality Transformation method with Outlier Consideration in Bitcoin Exchange Rate Analysis

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ABSTRACT

Bitcoin is the first decentralized peer-to-peer payment network that is powered by its users with no central authority or middlemen. The objective of this study is to evaluate the normality of data distribution for exchange rate of Bitcoin. The method implemented in this study is Shapiro-Wilk normality test including graphical approach namely box plot .Results show the data distribution of exchange rate for Bitcoin follows non-normal distribution. Therefore, the normality transformation is important to make sure the distribution of data follows normal distribution. The normal distribution is very crucial as one of the requirement for validity of statistical test.Normality tests are used to determine if a data set is well-modeled by a normal distribution and to compute how likely it is for a random variable underlying the data set to be normally distributed.This study implemented two-stages of outliers detection and deletion process.The final results shows the distribution of Bitcoin exchange rate with first difference is follow normal distribution with probability of 0.722.Result concluded the distribution of data after second stages of outlies deletion treatment shows high normal distribution characteristics. This finding concludes that Bitcoin data is highly volatile with existence of many outliers. The transformation process is highly important to make sure the Bitcoin data follows normal distribution that underlying critical assumption for statistical tests.

Key Words: *Crypto currency, Bitcoin, Exchange rate, Outliers, Normality test.*

1. INTRODUCTION

In year 2017, the price for Bitcoin increased sharply. This situation creates interest among investors to purchase Bitcoin. Bitcoin is a type of digital coins (cryptocurrency) which is not issued by any government, bank or organization. However, bitcoin is relying on cryptographic protocols and a distributed network of users to perform mining, storing, and transferring activities. Therefore, Bitcoin is not subject to regulation by central banks, does not the backing with intrinsic value and has a decentralisedexchange platform. This cryptocurrency gains its popularity as a means of settling e-commerce transactions (Ram, et al., 2016; Rees, 2014).

A Bitcoin transaction was introduced based on cryptographic, allowing two parties to transact directly with each other without the need for a trusted third party. This transaction are computationally impractical to reverse would protect sellers from fraud, and routine escrow mechanisms could easily be implemented to protect buyers (Nakamoto, 2009).Blockchains are a software protocol that underlie bitcoincryptocurrency in one sense, are nothing more than a modernizing information technology, but in another sense, are novel and disruptive (Yeoh, 2017).

Anonymity in Bitcoin, a peer-to-peer electronic currency system, is a complicated issue. Within the system, users are identified only by public-keys. An attacker wishing to de-anonymize users will attempt to construct the one-to-many mapping between users and public-keys, and associate information external to the system with the users. Bitcoin tries to prevent this attack by storing the mapping of a user to his or her public-keys on that user’s node only and by allowing each user to generate as many public-keys as required(Reid and Harrigan, 2013)

Nowadays, the demand for bitcoincryptocurrencywas increase due to development of e-commerce transaction. Although e-commerce is a new phenomenon of this modern age, it denotes the same connotation with that of traditional commerce in Islam, as e-commerce also involves contracts to exchange valuable assets or properties from one party to another to gain profit. Therefore, all e-commerce transactions must also conform to the requirements and principles of Islamic law of contract, which mainly aims at protecting interests and eliminating harms of parties involved in a transaction, thus promoting Islamic ethical code of commerce and business (Muhammad et al., 2013). After the crash of dot.com stock prices in March 2001, internet usage and e-commerce have continued to grow at a fast pace (Cheng et al., 2006). Consequently, bitcoincryptocurrency was introduced in 2009.

Therefore, it is important to analyze the behavior of Bitcoin price, in providing the accurate information to investors. In developing the statistical test for data of Bitcoin price, normality characteristics is one of the important element that need to be conform. This step is important to make sure all the finding from the statistical tests is valid. This study analyzed the data distribution of exchange rate from 1 Bitcoin to United States Dollar (USD). Outliers in the data distribution are detected using Shapiro-Wilk test and box plot graphical test. Next, the potential data as outliers are then deleted to improve the normality characteristics of the data.

2. LITERATURE REVIEW

Bitcoin is a cryptocurrency based on open-source software and protocols that operates in peer-to-peer networks as a private irreversible payment mechanism. The protocolallows cross-border payments, for large and small items, with little or no transactionalcosts. The bitcoin transactional system is often described as an anonymous system,although it might be more accurate to describe the system as one in which users can invoke privacy.The ledger of account for all Bitcoin transactions is public and distributed (Simser, 2015).

Bitcoin is a digital currency that unlike traditional currencies does not rely on a centralized authority. Instead Bitcoin relies on a network of volunteers that collectively implement a replicated ledger and verify transactions.The ability of cryptocurrencies to enable anonymous transactions allows users to trade virtual currency regardless of their geographic location, without revealing either the real-world source of their income or their own identity. Cryptocurrencies, such as Bitcoin, rely on a de-centralised system based on peer-to-peer public key addresses, rather than having a central regulating body, such as a financial institution or bank, which reviews and monitors transactions. Bitcoin currency is a monetary value that is accepted for payment purposes by persons other than the issuer, with the unit of account matching that of the physical currency (Bal, 2013). According to Christopher (2014) bitcoinoperates via a peer-to-peer (P2P) network. P2P networks are created when multiple individuals run the necessary software on their individual computers and connect to each other; P2P networks do not have a centralized website, server, or organizer.

Normal distribution is one of the key elements that determine type of statistical test that need to be performed (Robert Handerson, 2006). One of the methods is using data transformation. Data transformations are the application of a mathematical modification to a variable. There are a great variety of possible data transformations, from adding constants to multiplying, squaring or rising to a power, converting to logarithmic scales, inverting, taking the square root of the values, and even applying sine wave transformations (Osborne, 2002).There are a variety of reasons why researchers might want to employ data transformations. First, as many statistical procedures assume or benefit from normality of variables, data transformations can be employed to improve the normality of a variable's distribution. Authors of prominent statistical texts, such as Tabachnick and Fidell (2001), argue that researchers should "consider transformation of variables in all situations" unless there is a specific reason not to. Other reasons for utilization of data transformations involve equalizing variance (Bartlett, 1947), although this is less commonly the reason researchers turn to transformation.

If a researcher has a variable that is substantially non-normal, even if analyses utilized do not assume normality, improving normality can often enhance the outcome of analyses by reducing error. In fact, Tabachnick and Fidell (2001) explicitly state that, even when normality is not an issue, transformations can improve analyses. Zimmerman (1995, 1998) pointed out that non-parametric tests can suffer as much, or more, than parametric tests when normality assumptions are violated, confirming the importance of normality in all statistical analysis, not just parametric analyses. There are multiple options for dealing w ith non-normal data. First, the researcher must make certain that the non-normality is due to a valid reason. Invalid reasons include things such as mistakes in data entry, and missing data values not declared missing. These are simple to remedy. Outliers, scores that are extreme relative to the rest of the sample, are another reason for non-normality.

Testing whether a sample of observations comes from a Gaussian distribution is a problem that has attracted a great deal of attention over the years. This is not perhaps surprising in view of the fact that normality is a common maintained assumption in a wide variety of statistical procedures, including estimation, inference and forecasting procedures (Psaradakis and Vavra, 2017). In model building, a test for normality is often a useful diagnostic for assessing whether a particular type of stochastic model may provide an appropriate characterization of the data (for instance, non-linear models are unlikely to be an adequate approximation to a time series having a Gaussian one-dimensional marginal distribution). Normality tests may also be useful in evaluating the validity of different hypotheses and models to the extent that the latter rely on or imply Gaussianity, as is the case, for example, with some option pricing, asset pricing, and dynamic stochastic general equilibrium models found in the economics and finance literature. Other examples where normality or otherwise of the marginal distribution is of interest, include value-at-risk calculations (Cotter, 2007) and copula-based modeling for multivariate time series with the marginal distribution and the copula function being specified separately. Kilian and Demiroglu (2000) and Bontemps and Meddahi (2005) give further examples from economics, finance and econometrics where testing for normality is of interest.

This study focused on the data distribution of Bitcoin exchange rate. This study starts with analyzing the data stationary characteristics of Bitcoin exchange rate. In developing a stationary data, one of the methods introduced in this paper is by differencing with first order. Then, exchange rate data with differencing with first order is evaluated using Shapiro-Wilk test including graphical method namely box plot. The possible outliers that detected in this process will be evaluated and consider for deletion to increase the normality characteristics of data distribution. The purpose of this study is to confirm normality for data distribution that underlying all statistical tests.

3. RESEARCH METHODOLOGY

This study involved the statistical test in improving the data normality characteristics. The main objective for this study is detects outlier in data distribution. Then, this study developed deletion outliers to attain higher normality characteristics.

3.1 Probability density of normality distribution theory

A random variable *X* whose distribution has the shape of a normal curve is called a normal random variable.Figure 1 shows the normal curve for variable *X*. This random variable *X* is said to be normally distributed with mean μ and standard deviation σ if its probability distribution is given by Equation (1).

$$f\left(x\middle|\mu,\sigma^2\right)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{.. (1)}$$

where,

x is random variable;

μ is mean or expectation of the distribution;

σ is standard deviation;

π is approximately 3.14159; and

e is approximately 2.71828.

Figure 1 shows the normal data distribution curve. Empirical rule for normal distribution is stated as below:

(a) 68% of the data falls within one standard deviation of the mean.
(b) 95% of the data falls within two standard deviation of the mean.
(c) 99.7% of the data falls within three standard deviations of the mean.

The properties of normal distribution are described as:

(a) The mean, mode and median are all equal.
(b) The curve is symmetric at the center.
(c) Exactly half of the values are to the left of center and exactly half the values are to the right.
(d) The total area under the curve is 1.

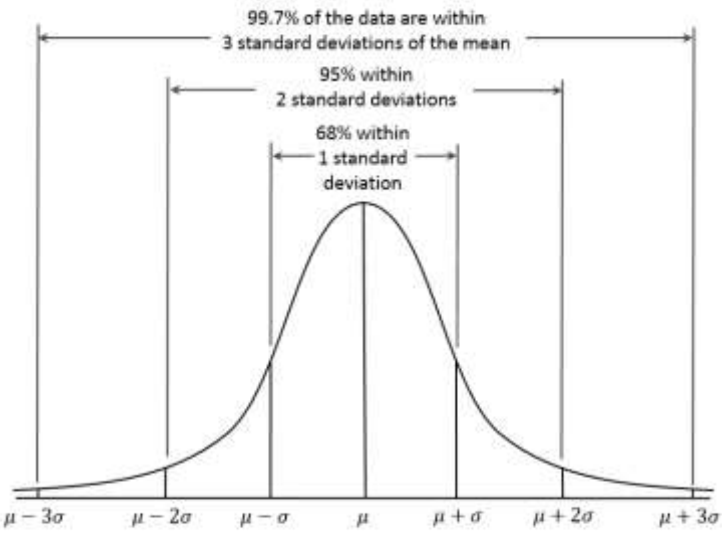

Figure 1: A normal distribution curve

3.2 Normality test using Shapiro-Wilk method

The normal distribution is useful because of the central limit theorem. In its most general form, under some conditions (which include finite variance), it states that averages of samples of observations of random variables independently drawn from independent distributions converge in distribution to the normal, that is, become normally distributed when the number of observations is sufficiently large. Physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have distributions that are nearly normal. Moreover, many results and methods (such as propagation of uncertainty and least squares parameter fitting) can be derived analytically in explicit form when the relevant variables are normally distributed.

The null-hypothesis of Shapiro-Wilk normality test is that the population is normally distributed. Thus, if the p-value is less than the chosen alpha level, then the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population. On the opposite side, if the p-value is greater than the chosen alpha level, then the null hypothesis that the data came from a normally distributed population cannot be rejected .

The Shapiro-Wilk test is a way to tell if a random sample comes from a normal distribution. The test gives you a W value. The W value larger than chosen alpha (0.05), will concludes the distribution of data follows normal distribution. The, if the data shows small values of W, it is indicate your sample is not normally distributed. The formula for the W value is:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)} \right)^2}{\sum_{i=1}^{n} \left(x_i - \overline{x} \right)^2} \quad \text{.. (2)}$$

where:

x_i is the value in the sample $\left(x_1, x_2, x_3, ..., x_n \right)$;

$x_{(i)}$ is the ordered sample values ($x_{(1)}$ is the smallest value in the sample);

$\overline{x} = \frac{\left(x_1 + x_2 + ... + x_n \right)}{n}$ is the sample mean;

a_i is constants that derived generated from the means, variances and covariances of the order statistics of a sample of size *n* from a normal distribution. The calculation of a_i is described in below equation.

$$\left(a_1, a_2, a_3, ..., a_n \right) = \frac{m^{\mathrm{T}} V^{-1}}{\left(m^{\mathrm{T}} V^{-1} V^{-1} m \right)^{1/2}} \quad \text{......................(3)}$$

where:

V isthe covariance matrix of those order statistics;

$$m = \left(m_1, m_2, m_3, ..., m_n \right)^{\mathrm{T}} \quad \text{... (4)}$$

Element in Equation (4) is represented as:

$m_1, m_2, m_3, ..., m_n$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution

The normality tests are supplementary to the graphical assessment of normality.The test rejects the hypothesis of normality when the p-value is less than or equal to 0.05. Failing the normality test allows you to state with 95% confidence the data does not fit the normal distribution. Passing the normality test only allows you to state no significant departure from normality was found.

3.3 Outliers data detection using Box-plot method

The box plot is a standardized way of displaying the distribution of data based on the five number summary: minimum (Lower extreme value limit), first quartile, median, third quartile, and maximum (upper extreme value limit). Figure 2 shows the box-plot diagram with outliers detection range. Meanwhile, Figure 3 shows the correlation between box-plot and normal distribution curve.

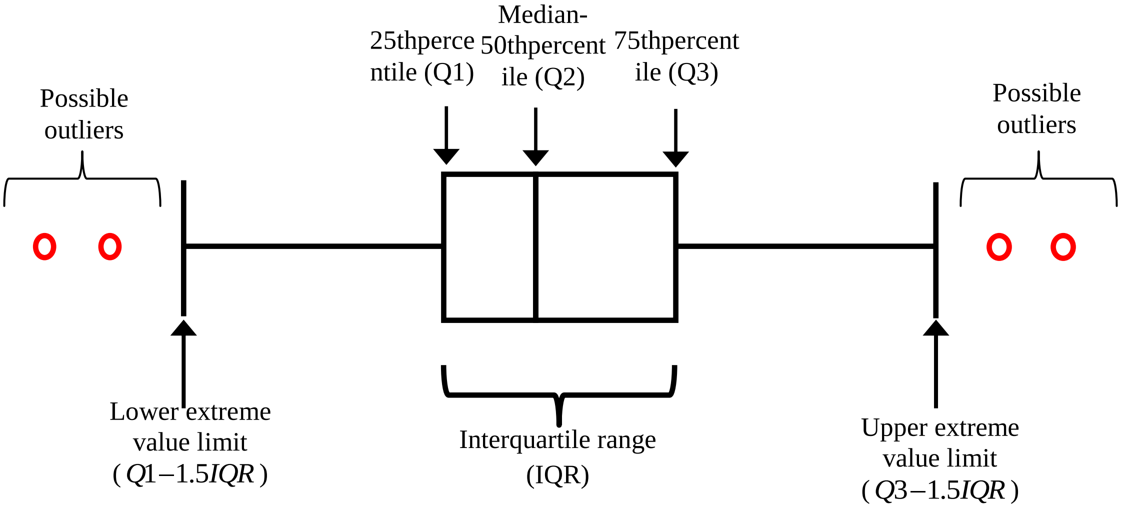

Figure 2: Box plot with outliers detection range

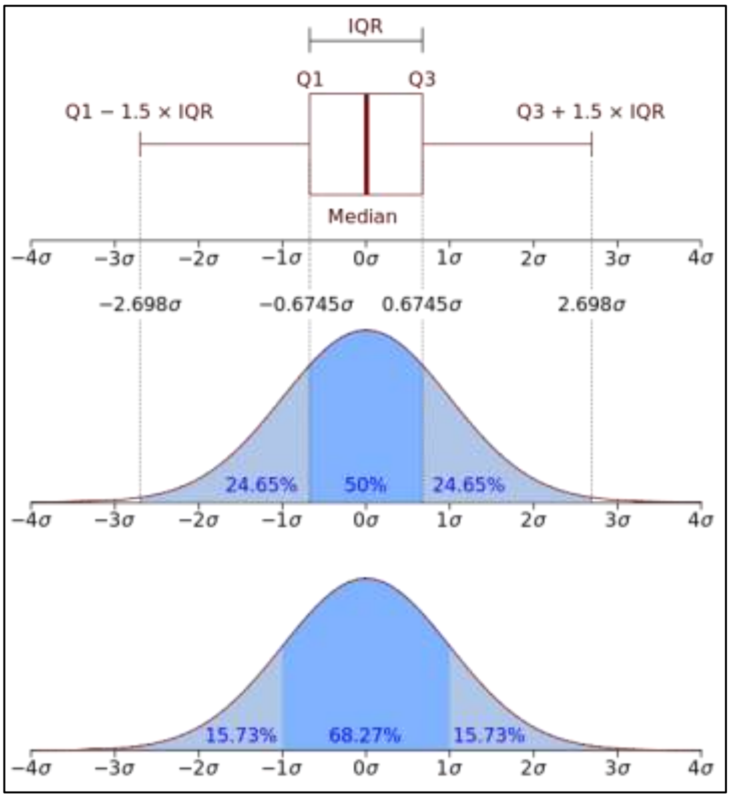

Figure 3: Box plot correlation with normal curve

4. RESULT AND DISCUSSION

This section describes the procedures of normality transformation for Bitcoin exchange rate. This study also implemented method of reducing outliers to achieve normal distribution of data. Data of Bitcoin exchange rate is collected from http://www.coindesk.com.

4.1 Normality test for Bitcoin exchange rate

This section explains the normality test that performed to Bitcoin exchange rate. Figure 4 shows the dynamic behavior of exchange rate with respect to United States dollar (USD). Data is collected from 23rd June 2017 until 23rd September 2017. The initial value of Bitcoin exchange rate is 2738.22 on 23rd June 2017.The minimum value of exchange rate is 1938.94 on 16th July 2017. The maximum value of exchange rate is 4950.72 on 1st September 2017.The final value of Bitcoin exchange rate is 3777.29 on 23rd September 2017.

Figure 5 shows the normal probability plot of Bitcoin exchange rate. Result shows the distribution of data is deviated from straight line. Therefore, departures from this straight line indicate departures from normality.The distribution of Bitcoin exchange rate follows non normal distribution.

Then, this study validated the distribution using Shapiro-Wilk normality test. Table 1 shows the p-value is 0.000. This probability value is less than 0.05. Therefore, the distribution of Bitcoin exchange rate is non-normal distribution.

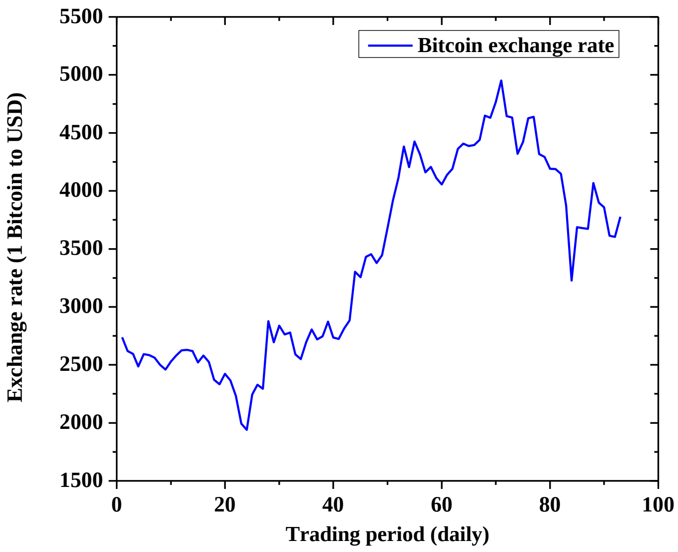

Figure 4: Bitcoin exchange rate

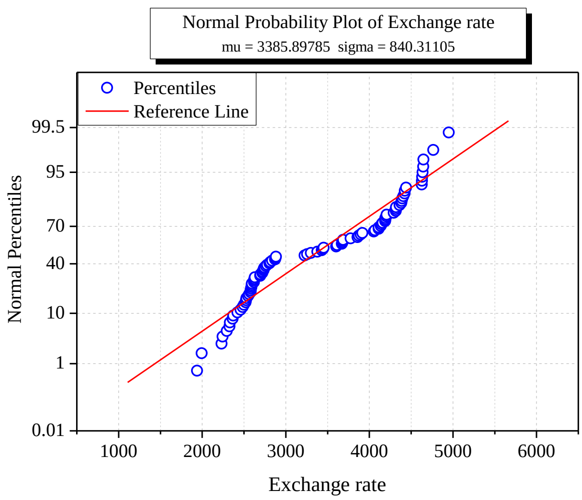

Figure 5: Normal probability plot of Bitcoin exchange rate

Table 1: Shapiro-Wilk normality test

	Shapiro-Wilk		
	Statistic	df	Sig.
Bitcoin exchange rate	.909	93	.000

4.2 Normality test for first difference ofBitcoin exchange rate

Section 4.1 shows the distribution of Bitcoin exchange rate follows non normal distribution. Therefore, this study performed the difference process of Bitcoin exchange rate with one level. Difference value of exchange rate is calculated using Equation (5).

$$\Delta ER_t = ER_t - ER_{t-1} \text{ ……………………………………….. (5)}$$

ΔER_t : Difference value of Bitcoin exchange rate at period *t*

ER_t : Bitcoin exchange rate at period *t*

ER_{t-1} : Bitcoin exchange rate at period *t-1*

Figure 6 shows the dynamic behavior of difference value forBitcoin exchange rate. The maximum value of first difference for Bitcoin exchange rate is 582.99 on 20th July 2017. The minimum value is -647.85 on 14th September 2017.

Then, this study checked the normality for data distribution. Figure 7 shows the normal probability plot for difference of Bitcoin exchange rate. Figure 7 shows there are data points that are deviated from the normality line. Therefore, the distribution of this sample becomes non normal distribution.

Then, the distribution of data is validated with the analytical of normality test. Table 2 shows the normality test performed to the data of the difference value for Bitcoin exchange rate. The probability value is 0.006. This value is less than 0.05. Therefore, the distribution of data is follows non normal distribution.

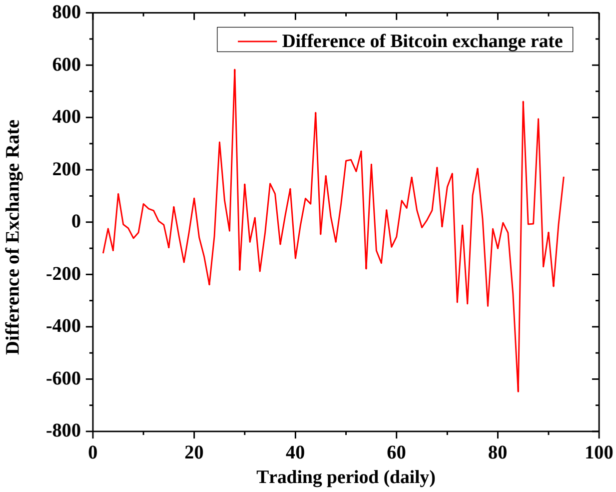

Figure 6: Bitcoin exchange rate

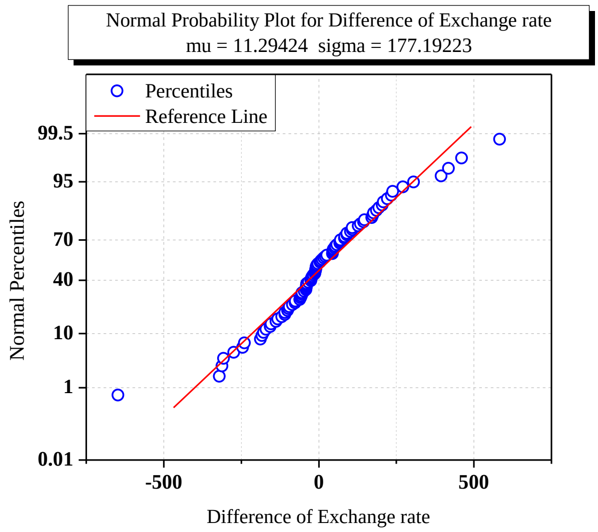

Figure 7: Normal probability plot for Difference exchange rate

Table 2: Shapiro-Wilk normality test

	Shapiro-Wilk		
	Statistic	df	Sig.
Difference of Bitcoinexchange rate	.960	92	.006

4.3 Outliers detection from data using graphical method (First stage outlier detection process)

In section 4.2, the distribution of first difference for Bitcoin exchange rate is follow non normal distribution. In evaluating the normality characteristics of the data distribution, this study performed graphical test to detect outlier data point.

Figure 8 shows the boxplot for data distribution of Bitcoin exchange rate with first difference. From the graphical analysis, there is one data point (84th observation period data) that located at less than 5th percentile. In addition, there are four data points (data observation 28, 44, 88, 85) that larger than 95th percentile. These five observations are classifying as outliers data points.

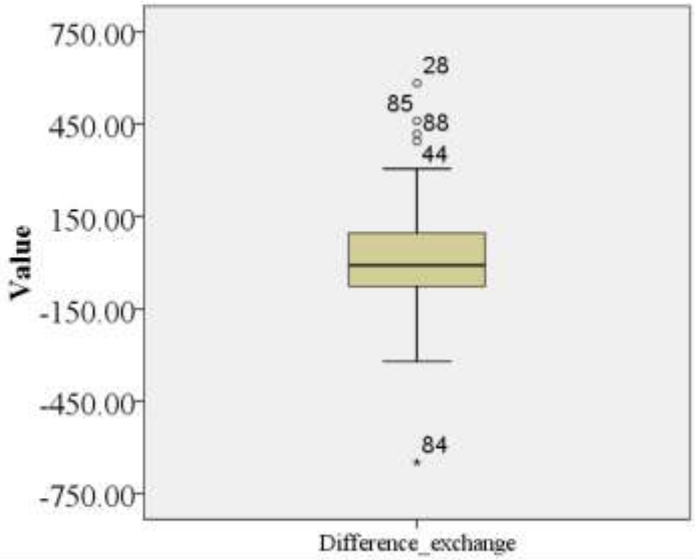

Figure 8: Boxplot for data distribution of Bitcoin exchange rate with first difference.

4.4 Normality test without Outliers data point(First stage outlier deletion process)

This section describes the normality test without outliers. Firstly, this study developed histogram for data distribution without outliers detected in section 4.3. Figure 9 shows histogram for data distribution of Bitcoin exchange rate with first difference (without outliers). Figure 9 shows the data distribution is close to normal distribution curve. Therefore, the distribution is follow normal distribution.

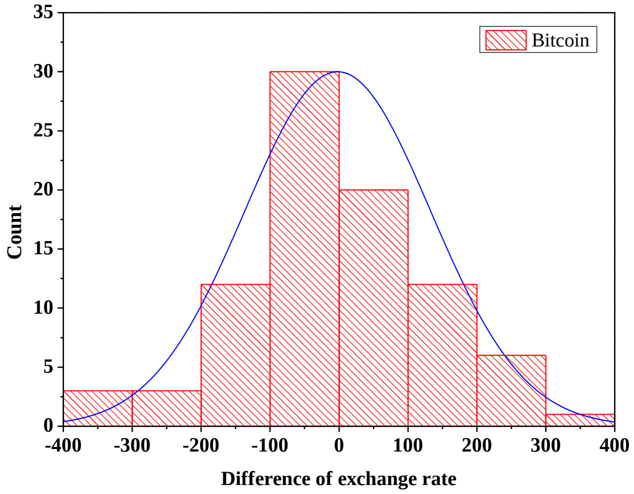

Figure 9: Histogram for data distribution of Bitcoin exchange rate with first difference (without outliers).

Next, this study developed the normal percentiles graph. Figure 10 shows the normal probability plot for data distribution of Bitcoin exchange rate with first difference (without outliers).Figure 10 shows most of the data points are near to the normality line. Therefore the distribution of the data is follow normal distribution.

Then, this study validated the graphical normality result using numerical approach. Table 3 shows the normality test for data distribution using Shapiro-Wilk method. The propability for Shapiro Wilk test is 0.551. This value is larger than 0.05. Therefore, the distribution of data follows normal distribution data. The method of deleting outliers in this study is effective in transforming the non-normal distribution to normal distribution.

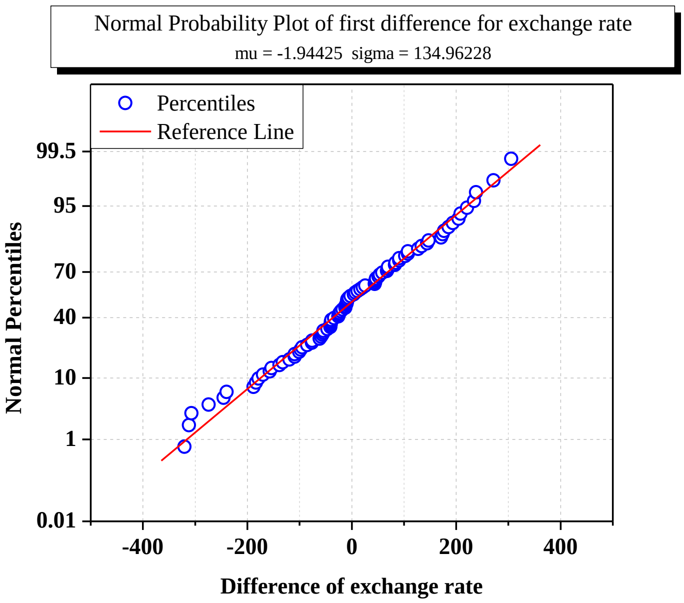

Figure 7: Normal probability plot without outliers

Table 3: Shapiro-Wilk normality test

	Shapiro-Wilk		
	Statistic	df	Sig.
Different of exchange rate (without outliers)	.987	87	.551

4.5 Outliers detection from data using graphical method (Second stage outlier detection process)

This study is achieving the normal distribution as stated in section 4.4. However, this study also re-examine again the distribution of data whether there is any outliers in the sample. The purpose of this process is to increase the probability of normality characteristics. The increment of normality probability contributes to reliable inferential result.

Figure 11 shows the boxplot for data distribution of Bitcoin exchange rate with first difference (second stage outlier detection process).This study detected one outlier in second stage of outlier detection process. The 78th data observation is the categorical as outlier that near to 5th percentile. This outlier is needed to be omitted in obtaining better normality characteristics for data distribution.

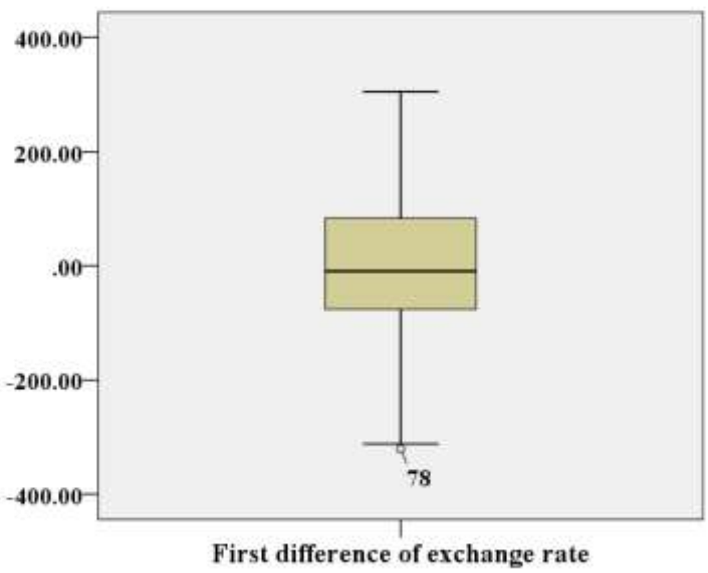

Figure 11: Boxplot for data distribution of Bitcoin exchange rate with first difference (second stage outlier detection process).

4.6 Normality test without Outliers data point (Second stage outlier deletion process)

This section describes the normality test after the second treatment of outliers. Figure 12 shows histogram for data distribution of Bitcoin exchange rate in first difference. Data selected in this histogram is data without outliers. The histogram developed after the second deletion process of outliers. Figure 12 shows the distribution of data observation is close to normality line. Therefore, the distribution of data after second treatment of outliers is follow normal distribution.

Next, this study developed the normal percentiles graph. Figure 13 shows the normal probability plot for data distribution of Bitcoin exchange rate with first difference (after second outlier deletion treatment). Figure 13 shows most of the data points are near to the normality line. Therefore the distribution of the data is follow normal distribution.

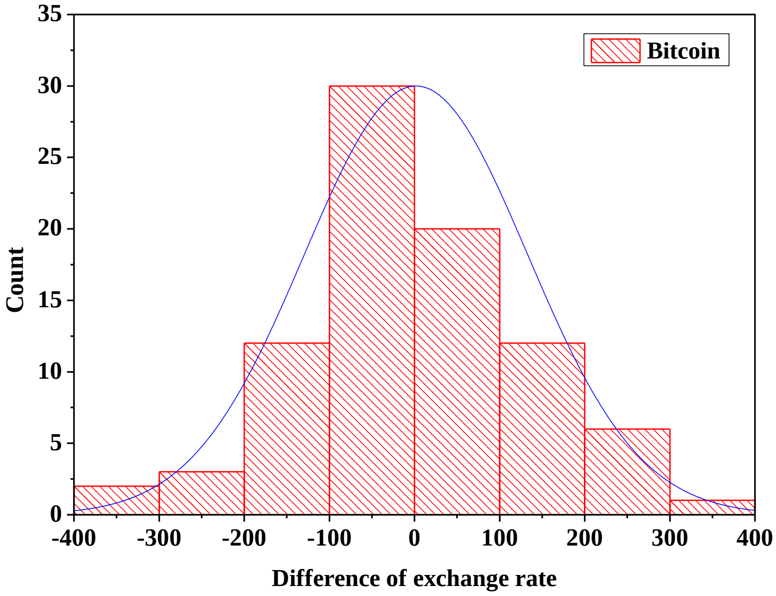

Figure 12: Histogram for exchange rate with first difference (after second stage of outliers deletion).

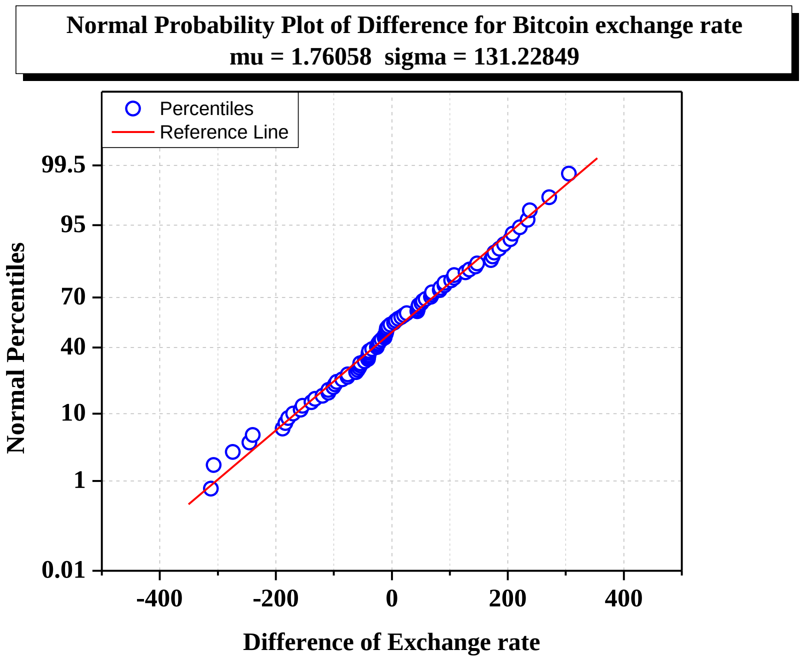

Figure 13: Normal probability plot for difference of Bitcoin exchange rate without outliers

Then, this study validated the graphical normality result using numerical approach. Table 4 shows the normality propability for Shapiro Wilk test is 0.722. This value is larger than 0.05. Therefore, the distribution of data follows normal distribution data. The method of second stage for deleting outliers in this study is effective in transforming the non-normal distribution to normal distribution and increasing the distribution data to become highly normality distribution.

This study performed the box-plot analysis to examine any existance of outliers. Figure 14 shows Boxplot for data distribution of Bitcoin exchange rate with first difference (after second stage outlier detection process). Figure 14 concludes data distribution follows normal distribution without any existence of outliers.

Table 4: Shapiro-Wilk normality test

	Shapiro-Wilk		
	Statistic	df	Sig.
Different of exchange rate (without outliers after second stage detection)	.989	86	.722

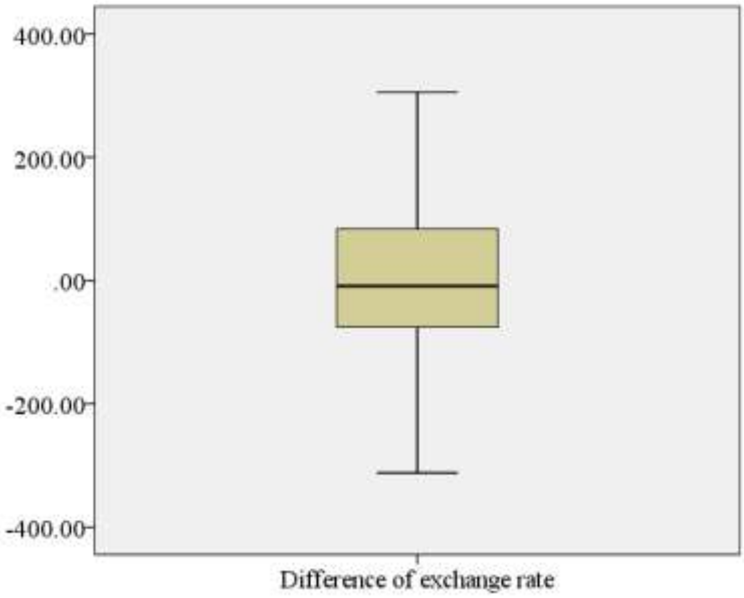

Figure 14: Boxplot for data distribution of Bitcoin exchange rate with first difference (after second stage outlier detection process).

5. CONCLUSION

The objective of this study is to examine the normality of the Bitcoin exchange rate. Data selected in this study is Bitcoinexchange rate with respect to United States dollar (USD). Data is collected from 23rd June 2017 until 23rd September 2017.

a. The distribution of Bitcoin exchange rate follows non normal distribution.This study validated the distribution using Shapiro-Wilk normality test. Calculation shows the p-value is 0.000. This probability value is less than 0.05. Therefore, the distribution of Bitcoin exchange rate is non-normal distribution.

b. In transforming the non-normal distribution to become normal distribution, this study calculated the difference of exchange rate. Then, this study examines the distribution of data using the analytical of normality test. Result shows the probability value is 0.006. This value is less than 0.05. Therefore, the distribution of data is follows non normal distribution.

c. The non-normal distribution in difference of exchange rate is contributed mainly because of the existence of outliers. Therefore, this study performed first stage of outlier detection and outlier deletion process. Result show the probability value is increase to become 0.551. This probability value is larger than 0.05. This concludes the distribution data of the difference of exchange rate follows normal distribution.

d. In improving the normality characteristics, this study performed the second outlier detection and deletion process. Result shows the probability value increases to 0.722. This value shows the current distribution data set is high normally distributed.

Normality characteristics are main assumption in any statistical test. Therefore, this finding is important to economists and statistician in evaluating the performance of Bitcoin exchange rate using statistical test.

Further research of this study can be extending to find the factors that contribute to the outliers existence in the data set.

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