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Vibration Control of Cantilever Beam with Multiple Cracks

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ABSTRACT

Cracks often develop in structural members and cracking can cause serious durability issues as well as structural damage. Cracks influence dynamic characteristics of the structural members and have been the subject of many investigations. In the present work, a numerical study using finite element method is performed to investigate the transverse free vibration response of a cracked isotropic cantilever beam using ANSYS. A parametric study is also carried out to assess the influence of crack depth ratio, location of cracks and number of cracks on the first three natural frequencies of the beams. Vibration control studies are also carried out. The results can be utilised to locate cracks and cracking intensity within a remote or massive structure by real-time monitoring of ambient vibration data.

Key Words: *Cracking, Dynamic Characteristics, Free Vibration, Vibration Control, ANSYS, Natural Frequencies.*

I. INTRODUCTION

Cracking is inevitable in civil engineering structures, and if exceeded the limits, can cause structural damage as well as adversely affect the serviceability and durability. The presence of cracks in the structural components can have a significant influence on the dynamic properties of a structure. Early identification of cracking in structural members is essential in engineering practice, and is achievable by identifying the change in dynamic properties of the members. Vibration control is defined as a technique in which the vibration of a structure is reduced or controlled by applying counter force to the structure that is appropriately out of phase but equal in amplitude to the original vibration

Many studies on dynamic behaviour of cracked beams have been reported in the literature. Location, depth and intensity of cracking can be reckoned by interpreting the changes in natural frequencies [5]. Behzad et al. [3] presented a study on vibration analysis of a cracked beam, wherein, the equations of motion and corresponding boundary conditions for bending vibration of a beam with an open edge crack was developed by implementing the Hamilton principle. Chondros and Dimarogonas [4] developed a continuous cracked bar vibration model for the lateral vibration of a cracked Euler-Bernoulli cantilevered beam with an edge crack. Kisa and Brandon [8] used a bilinear stiffness model for taking into account the stiffness changes of a cracked beam in the crack location. Pawar and Sawant [11] developed the study of the vibration analysis of cracked cantilever beam subjected to free and harmonic excitation at the base. Zheng [18] studied the natural frequencies and mode shapes of a cracked beam, employing the finite element method. Zsolt [19] analysed quasi periodic opening and closings of cracks for vibrating reinforced concrete beams by laboratory experiments and by numeric simulation. Giannini *et al.,* [6] proposed a methodology for the identification of breathing cracks in beams able to detect simultaneously the location and the depth of the damage. Nicola and Ruotolo [14] introduced a technique aimed to evaluate the dynamic response of a beam with multiple breathing cracks to an applied sinusoidal force. Rezaee and Fekrmandi [13] investigated the free nonlinear vibration behaviour of a cracked cantilever beam, both theoretically and experimentally. Ruotolo *et al.,* [11] analysed the vibration response of a cracked cantilevered beam to harmonic forcing. Khalatkar *et al.,* [7] investigated the influence of actuator location and configurations in order to identify the optimal configuration of the actuators for selective excitation of the mode shapes of the of cantilever plate structure. Narayanan and Balmurugan [9] studied active vibration control performance of piezolaminated structures using classical control methods. Vasques and Rodriguez [17] investigated numerical study concerning the active vibration control of smart PZT beams.

The objective of this paper is to present the effect of depth and position of cracks on the natural frequencies of transverse vibration of isotropic cantilever beams. Numerical model of the beam is developed using finite element method by employing ANSYS package. The developed finite element model is validated using analytical solution of the field equation of beams. A detailed parametric study is also performed in order to assess the effect of variation of crack location, crack depth ratio and number of cracks on dynamic properties of the beam. Vibration control is done on the same model and studies are done by varying thickness voltage and damping constants of PZT.

II. MATHEMATICAL MODEL

A cantilever beam with and without a transverse edge crack which is clamped at left end, free at the right end is considered as shown in figure 1.

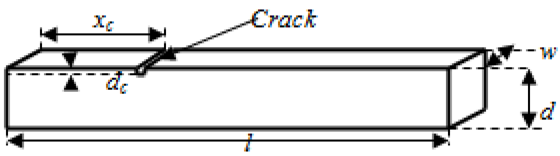


Fig. 1 Beam model with one crack

The general equation for natural frequency (ω_n) in transverse vibration of a cantilever isotropic beam of the constant rectangular cross section having a length l, width b, depth d, crack length x_c crack depth d_c, A is the cross section area ($l \cdot b$), EI is the flexural rigidity, ρ is the density of material and t is the time is given by [1],

$$\omega_n = C\sqrt{\frac{EI}{\rho A l^4}} \tag{1}$$

where, C is the constant depending mode of vibration, having values 0.56, 3.52 and 9.82 for first three modes, respectively [12]. Due to presence of a crack, I in eq. (2) can be replaced with reduced moment of inertia (I_r), which can be found by the following relation [15]:

$$I_r = I - I_c \tag{2}$$

where, I_C is the moment of inertia of cracked part of the beam. In order to identify the intensity of cracking, one can define crack depth ratio (ζ) as,

$$\zeta = \frac{d_c}{d} \qquad (3)$$

III. NUMERICAL FORMULATION

A 20-noded three-dimensional structural brick element, SOLID186 [20] was used in the present numerical model. The cracks in the beams are modelled as the part of the continuum in the present formulation. A fine mesh was provided in the cracked region of the beam so as to accommodate the physical discontinuity due to cracks in the continuum. The mesh configuration at the cracked edge of beam is shown in figure 2.

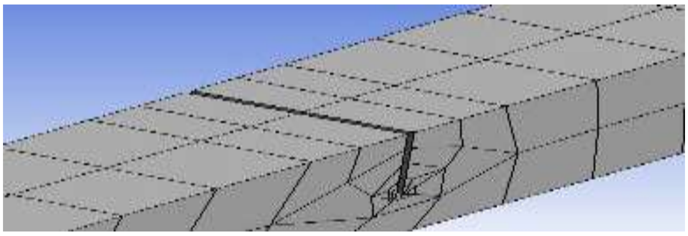


Fig. 2 Finite element discretisation at the cracked part of a beam

IV. RESULT AND DISCUSSION

IV. 1 FREE VIBRATION RESPONSE

The deflection control of cracked beam was explained in the following section, with a validation and parametric studies.

IV.1.1 Validation

In order to validate the present numerical model, two different cases of an isotropic beam of dimensions 800 mm × 20 mm × 20 mm, with one as well as two cracks were considered.

Table 1 Modal frequencies of 800 mm × 20 mm × 20 mm beam with single and double cracks

Source	Modal frequencies of beam with single crack (Hz)			Modal frequencies of beam with double cracks (Hz)		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
Exact (present)	26.166	164.472	458.839	26.208	164.736	459.576
Numerical (present)	26.192	164.292	459.284	26.165	163.627	459.202
Numerical [16]	26.123	164.092	459.603	26.095	163.322	459.601

The beam was modelled in ANSYS and first three natural frequencies were extracted, the result of which is presented in table 1. The other properties are as follows: Young's modulus, E = 2.1 x 10^{11} N/mm^2, ρ = 7800 kg/mm^3, poisons ratio, v = 0.35, crack width = 1 mm, first crack depth d_1 = 2 mm and located at x_1 = 120 mm from the clamped end, second crack depth d_2 = 3 mm and position x_2 = 400 mm. The result from numerical analysis is found to be in excellent agreement with the exact solution and the solution available in the literature [16].

IV.1.2 Parametric Studies

In order to assess the influence of various parameters on the modal frequencies of cracked cantilever beam, a parametric study was carried out. An isotropic beam was considered is shown in figure 3 with the following properties: length l = 1000 mm with a rectangular cross-section with width b = 25 mm and height h = 25 mm, E= 2.07x10^{11} N/mm^2, ρ = 7850 kg/mm^3 and ν = 0.3. The PZT bimorph beam having length, l = 1000 mm and breadth, b = 25 mm is used for vibration control studies. Modal analysis of the beam was performed for varying crack depth ratio, crack location and number of cracks with varying location from the clamped end.

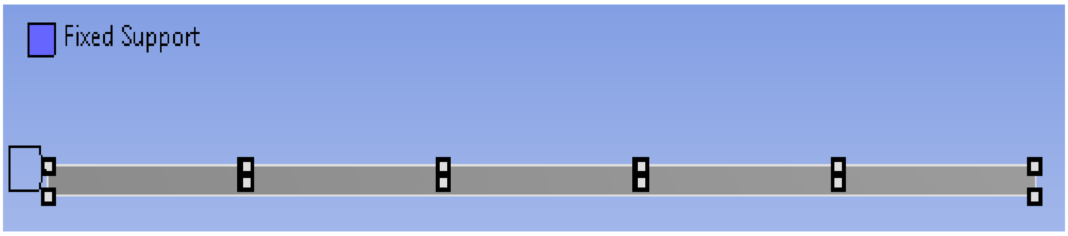


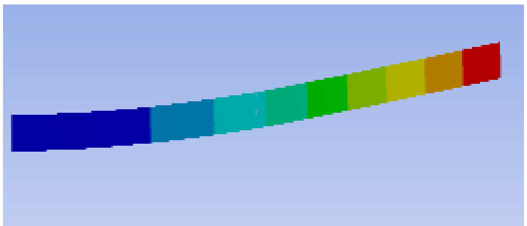
Fig. 3 Finite element model of beam with multiple cracks

(i) Effect of crack depth ratio (ζ)

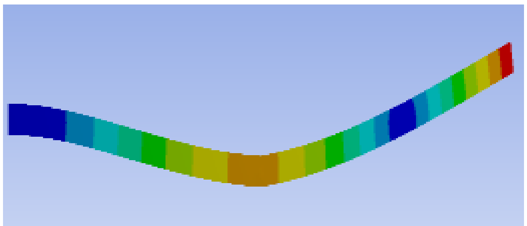
In order to study the effect of change in crack depth on the frequencies, the beam is given a single crack of width 1 mm at the midspan with different values of ζ. The table 2 presents the frequency variation of the beam with different ζ values. It can be interpreted from table 2 that, as ζ increases, the first three frequencies of the beam decrease. However, it can be noticed that the second and third modal frequencies have significant reduction. This can be attributed to the fact that, the beam has the crack at the midspan. The figure 4 shows the first three mode shapes of with varying crack depth ratio.

Table 2 Modal frequencies of 1000 mm × 25 mm × 25 mm beam for different crack depth ratios

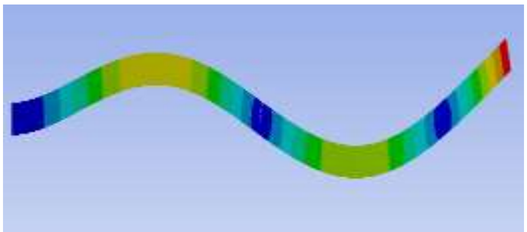
Crack depth ratio (ζ)	Modal frequencies of the beam (Hz)		
	Mode 1	Mode 2	Mode 3
Uncracked	20.41	127.47	355.28
0.2	20.35	126.21	355.27
0.4	20.19	121.96	355.23
0.6	19.69	111.69	355.12



(a)



(b)



(c)

Fig. 4 First three mode shapes of 1000 mm × 25 mm × 25 mm beam with varying crack depth ratio

(ii) Effect of crack location

The beam with different crack locations from the support, viz., 0.2 *l*, 0.4 *l*, 0.6 *l* and 0.8 *l* was analysed with ζ value 0.6, in order to identify the effect of crack locations on modal frequencies. The first three modal frequencies are presented in table 3. It is evident from the table 4.11 that, the first modal frequency is more affected by the crack at 0.2 *l* from the support. Similarly, crack location 0.4 *l* as well as 0.6 *l* induces more reduction in second frequency. Furthermore, the third mode is more influenced by cracks located at 0.8 *l*.

Table 3 Modal frequencies of 1000 mm × 25 mm × 25 mm beam with single crack for different crack locations

Crack location	Modal frequencies of the beam (Hz)		
	Mode 1	**Mode 2**	**Mode 3**
Uncracked	20.41	127.47	355.28
0.2 *l*	18.59	127.33	341.64
0.4 *l*	19.88	118.2	340.18
0.6 *l*	20.07	112.1	333.41
0.8 *l*	20.39	123.8	318.20

(iii) Effect of multiple cracks on different crack location

To study the influence of multiple cracks at different locations on the free vibration response of the beam, four cracks with ζ value 0.6 were induced in the numerical model of the cantilever beam as shown in figure 5. Modal frequencies were extracted and the results of which are presented in table 4.

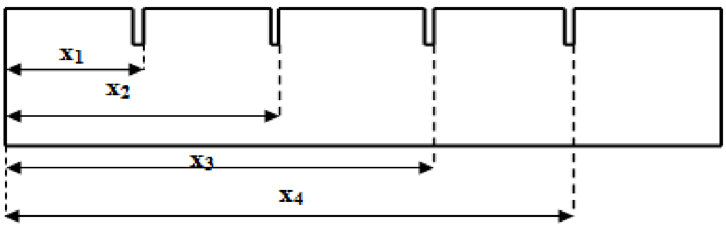


Fig. 5 Beam model with 4 cracks

It can be observed from table 4.12 that, the three modes is adversely affected by cracks and frequency gets shifted greatly when comparing to uncracked beam. Rigorous studies are necessary to characterise the behaviour of beam induced with multiple cracks with varying the crack locations. In order to find the amplitude of vibration harmonic analysis is done on beam with four cracks.

Table 4 Modal frequencies of 1000 mm × 25 mm × 25 mm beam with four cracks for different crack locations

Crack locations	Modal frequencies of the beam (Hz)		
	Mode 1	**Mode 2**	**Mode 3**
Uncracked	20.41	127.47	355.28
0.2 *l*, 0.4 *l*, 0.6 *l*, 0.8 *l*	16.63	104.64	302.35

Harmonic analysis is carried out after free vibration analysis. The fundamental frequency of four cracked beam i.e., 16.62 Hz is excited on 16 Hz to 17 Hz for an iteration of 10 times and amplitude variation is observed. The figure 6 shows the amplitude variation of vibration obtained for harmonic analysis.

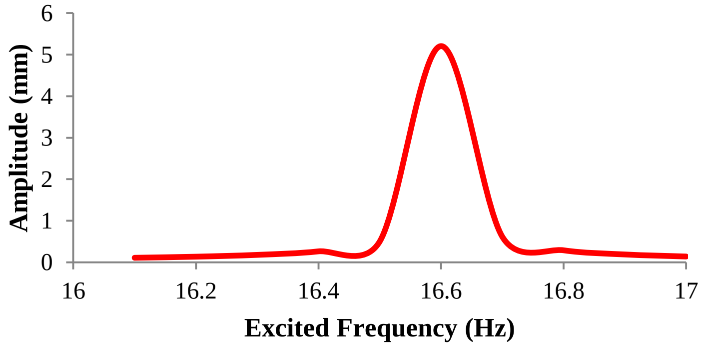


Fig. 6 Amplitude variation on excited frequency

4.5 VIBRATION CONTROL

4.5.1 Validation

The present numerical model is validated using an isotropic cantilever beam, of having length, l = 380 mm, breadth, b = 100 mm and height, h = 2 mm with PZT on top and bottom layer of l = 100 mm, b = 80 mm and height, h = 1 mm is considered is shown in figure 7. The elastic properties of the PZT material are given in table 5.

Table 5 Material properties of PZT

Anisotropic material coefficients (GPa)		
C_{11} = 132		C_{33} = 115
C_{12} = 73		C_{44} = 26
C_{13} = 71		C_{66} = 30
PZT stress coefficient (C m^{-2})		
e_{31} = -4.1	e_{33} = 14.1	e_{15} = 10.5
Dielectric constant (F m^{-1})		
$\epsilon_{11} = 7.124 \times 10^{-9}$	$\epsilon_{22} = 7.124 \times 10^{-9}$	$\epsilon_{33} = 5.841 \times 10^{-9}$

The other properties are as follows: For the steel beam Young's modulus, E = 2.07 x 10^{11} N/mm^2, density, ρ = 7870 kg/mm^3, poisons ratio, ν = 0.30 and for PZT, Young's modulus, E = 99 x 10^9 N/mm^2, density,

ρ = 7500 kg/mm^3, poisons ratio, ν = 0.31. The beam element of Solid 186 is used for steel beam, for PZT solid 226 and for adhesive contact 174 is used for the validation study.

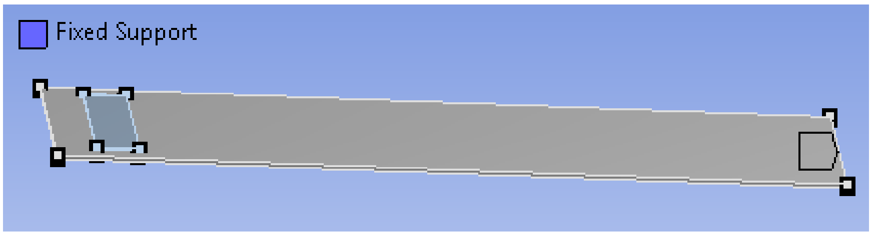


Fig. 7 Finite element model of bimorph beam with support and voltage location

The beam was modeled in ANSYS and modal frequencies were extracted, the result of which is presented in table 6.

Table 6 Modal frequencies of 380 mm × 100 mm × 2 mm beam with PZT

Source	Modal frequencies of beam with PZT (Hz)				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Numerical (present)	6.68	46.30	50.88	139.76	169.43
Numerical [2]	6.62	47.52	50.80	140.53	170.56

The result from numerical analysis is found to be in excellent agreement with the solution available in the literature [2].

4.5.2 Parametric Studies

In order to control the deflection of cracked cantilever beam, a parametric study was carried out. An isotropic beam was considered with the following properties: length l = 1000 mm with a rectangular cross-section with width b = 25 mm and height h = 25 mm, E= 2.1x10^{11} N/mm^2, ρ = 7850 kg/mm^3 and ν = 0.3. Deflection control of the beam was performed for varying thickness and voltage applied on PZT materials. The PZT bimorph beam having length, l = 1000 mm and breadth, b = 25 mm and having a density, ρ = 7500 Kgmm^{-1} is used for deflection control studies. The elastic properties of the PZT material are given in table 7 and the model details are shown in figure 8.

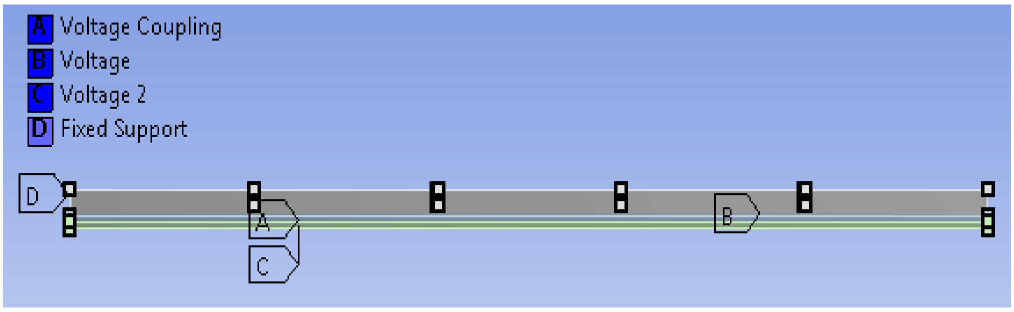


Fig. 8 Finite element model of cracked beam with PZT

Table 7 Material properties of PZT

Anisotropic material coefficients (GPa)		
C_{11} = 132		C_{33} = 115
C_{12} = 73		C_{44} = 26
C_{13} = 71		C_{66} = 30
PZT stress coefficient (C m^{-2})		
e_{31} = -5.6	e_{33} = 15.8	e_{15} = 12.3
Dielectric constant (F m^{-1})		
$\in_{11} = 1.730 \times 10^{-8}$	$\in_{22} = 1.730 \times 10^{-8}$	$\in_{33} = 1.70 \times 10^{-8}$

(i) Effect of variation of thickness on PZT material

The study was conducted to control the vibration by effect of variation of thickness on PZT material and variation was plotted. From the table 8 it was clear that as the thickness increases the modal frequency also increases and vibration control is greatly achieved on 6mm thickness. The typical thickness of 6mm is taken for the further study.

Table 8 Modal frequencies of 1000 mm × 25 mm × 25 mm beam with PZT on varying thickness

Thickness (mm)	Modal frequencies of the beam (Hz)		
	Mode 1	Mode 2	Mode 3
Uncracked	20.41	127.47	355.28
4 cracked	16.62	102.77	282.01
0	17.46	108.12	298.44
2	17.88	110.52	303.39
3	18.73	115.75	319.41
4	19.67	121.57	333.39
5	20.69	127.87	351.18
6	21.76	134.45	369.14

(ii) Effect of variation of applied voltage on PZT material

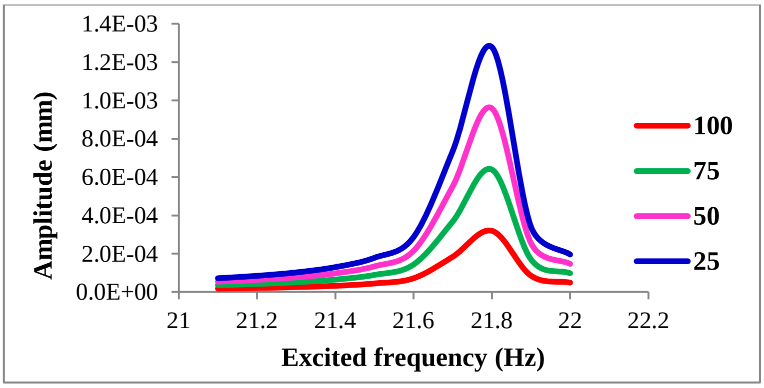


Fig. 9 Control of vibration on varying voltage on PZT material

In order to study effect of variation of voltage applied on PZT material and to control the vibration, harmonic analysis was done on 6mm thickness and variation was plotted. From the Figure 9 it was clear that as the voltage increases the amplitude of vibration decreases and can be controlled to nearly zero amplitude.

(iii) Effect of variation of damping on PZT material

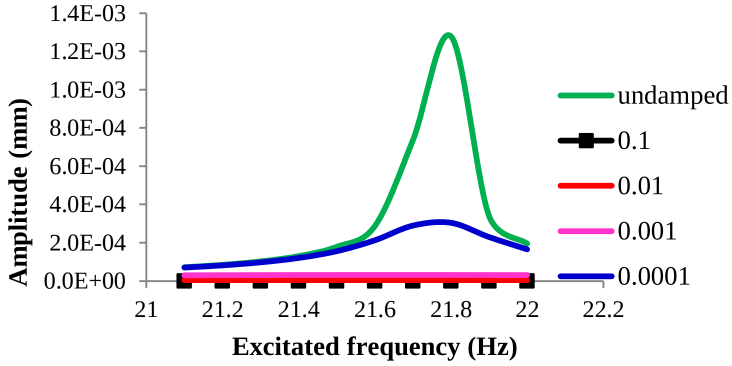


Fig. 10 Control of vibration on varying damping constants on PZT

Rigorous studies are needed on vibration control studies so variation on varying the damping constants. From the figure 10 it was clear that as the damping increases amplitude of vibration decreases thus the vibration control can be effectively achieved.

VI. CONCLUSIONS

Free vibration analyses of isotropic cantilever beam with multiple cracks are presented in this paper. The first three modes of each of the cases, viz., no cracks, one crack, two cracks and three cracks, are extracted, compared and interpreted. The study revealed that, if the cracks are concentrated near to the support of the cantilever beam, the first frequency gets shifted. Similarly, if the cracks are induced near to the midspan, the second mode is mainly affected. Similar behaviour can be observed with the third mode when the cracks are located near to the tip. Changes in the frequencies are observed to be enhanced when the crack depth is increased. This study proves that the modal frequencies are altered to change in depth of the crack and location of cracking in a beam. Active vibration control and deflection control can be greatly achieved using PZT materials. In order to control vibration it is advisable to optimize the thickness, voltage and damping and for deflection control thickness, length and voltage of PZT materials should be optimized. This study can be further extended in the area of Structural Health Monitoring (SHM), so as to predict the location and extend of damage in a remote structure or structural member.

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